A GPU-based Genetic Algorithm for the Set Cover Problem

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Set Cover Problem (SCP)

- A classic combinatorial optimization problem
- Can be framed as a graph analytics problem
  - Given a graph *G* with vertices *V* and weighted hyperedges ("sets") *E*, where *G = (V, E)*
  - Find a subset of hyperedges ("sets") whose union "covers" all vertices and minimizes the weight

Modeling an SCP as a Graph Problem

- Find a subset of hyperedges ("sets") whose union "covers" all vertices and minimizes the weight

$$Weights = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \end{pmatrix}$$

$$Incidence = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Modeling an SCP as a Graph Problem

- Find a subset of hyperedges ("sets") whose union "covers" all vertices and minimizes the weight

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$$Incidence = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Sets	1	2	3	4	5

Modeling an SCP as a Graph Problem

- Find a subset of hyperedges ("sets") whose union "covers" all vertices and minimizes the weight

$$Weights = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \end{pmatrix}$$

$$Incidence = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Weight = 9

Modeling an SCP as a Graph Problem

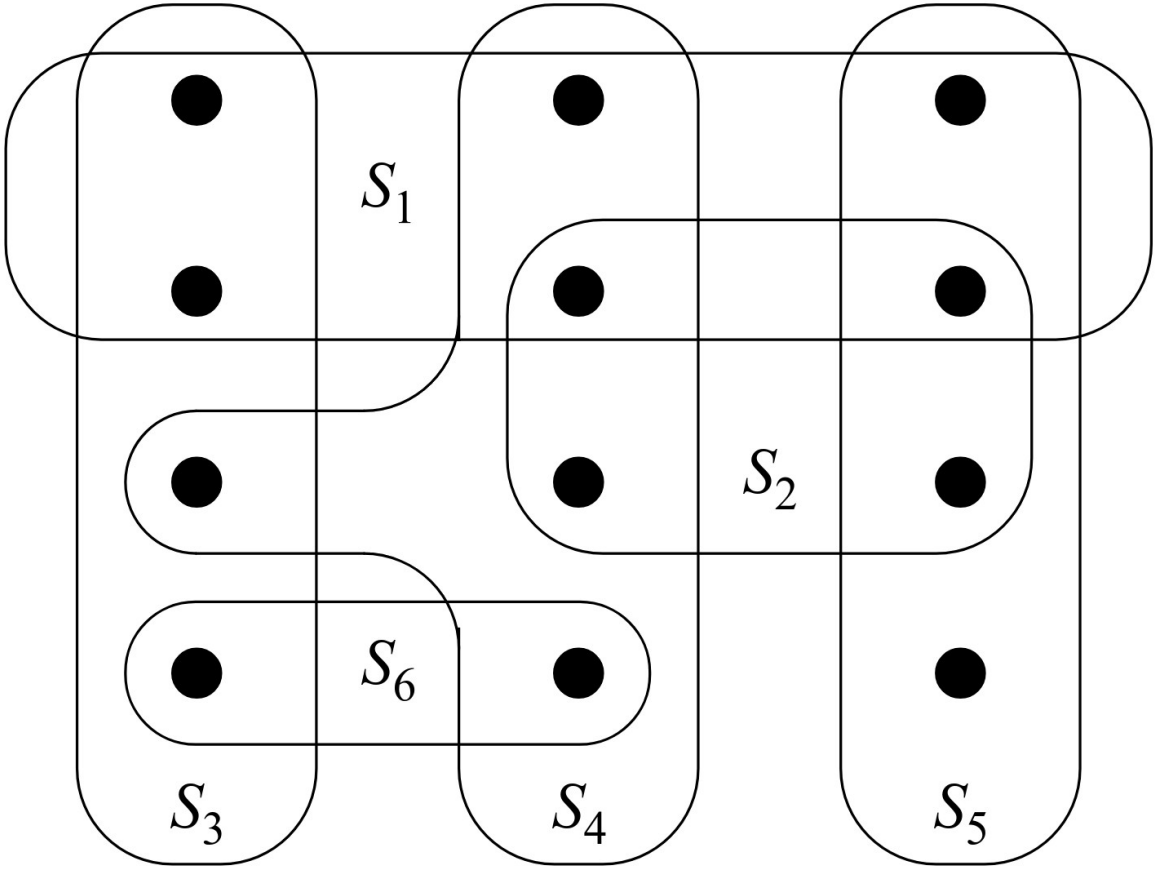
- Find a subset of hyperedges ("sets") whose union "covers" all vertices and minimizes the weight

$$Weights = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \end{pmatrix}$$

$$Incidence = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Weight = 7

Visualizing SCP With a Graph



Set Cover Problem (SCP)

- NP-Hard
- Variations of SCP
  - Non-Unicost vs. Unicost
  - Partial Cover
  - Conflicts on Sets
  - Small Neighborhood Property
  - and more…

SCP Applications

- Facility Location Selection
- Aircraft Crew Staffing
- Political Zoning
- Truck Transport
- Materials Management
- and more…

SCP & Linear Programming (LP)

Modeling an SCP Problem as LP

$$minimize \quad \sum_{j=1}^{n} c_j x_j$$

$$s.t. \quad x_j \in (0,1) \qquad j = 1, \ldots, n$$

Will explain shortly...

$$\sum_{j=1}^{n} a_{ij} x_j \geq 1 \qquad i = 1, \ldots, m$$

Linear Programming (LP) Example

Modeling an LP Problem

- Two bookshelf models x_1 and x_2 each require raw material and machine time
- x_1
  - $2 profit per unit
- x_2
  - $4 profit per unit

$$P = 2\,x_1 + 4\,x_2$$

Modeling an LP Problem

- Two bookshelf models x_1 and x_2 each require raw material and machine time
- x_1
  - 3 m^2 of material
- x_2
  - 4 m^2 of material
- Raw material is limited
  - 1700 m^2 of material per week

$$3\,x_1 + 4\,x_2 \leq 1700$$

Modeling an LP Problem

- Two bookshelf models x_1 and x_2 each require raw material and machine time
- x_1
  - (1 / 5) hours
- x_2
  - (1 / 2) hours
- Machine time is limited
  - 160 machine hours per week

$$\frac{1}{5} x_1 + \frac{1}{2} x_2 \leq 160$$

Modeling an LP Problem

- Two bookshelf models x_1 and x_2 each require raw material and machine time
  - Profit ($)
  - Material (m^2)
  - Machine Time (hrs)

$$P = 2x_1 + 4x_2$$

$$3x_1 + 4x_2 \leq 1700$$

$$\frac{1}{5}x_1 + \frac{1}{2}x_2 \leq 160$$

Modeling an LP Problem

$$P = 2\,x_1 + 4\,x_2$$

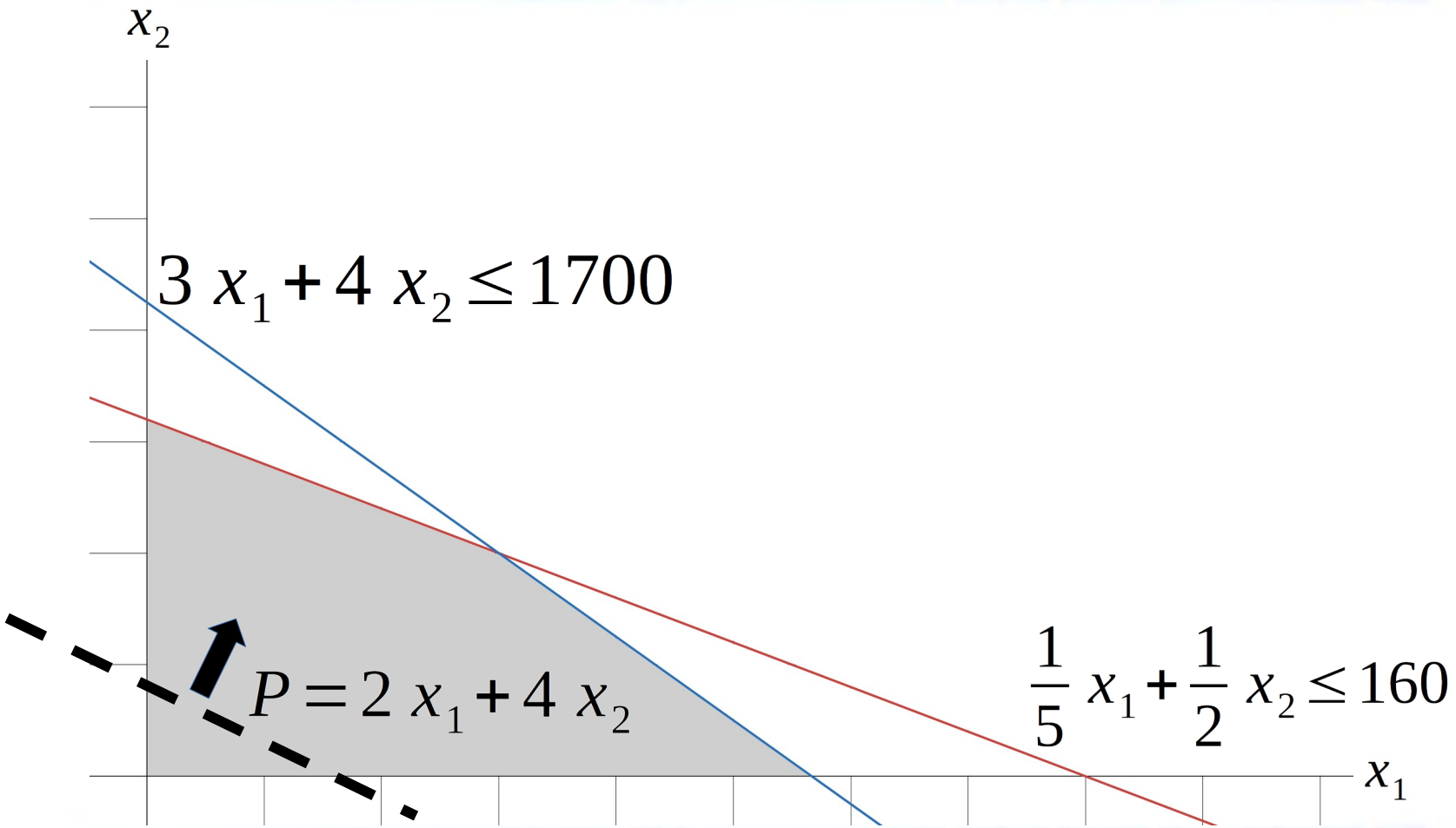
"Objective Function"

$$3\,x_1 + 4\,x_2 \leq 1700$$

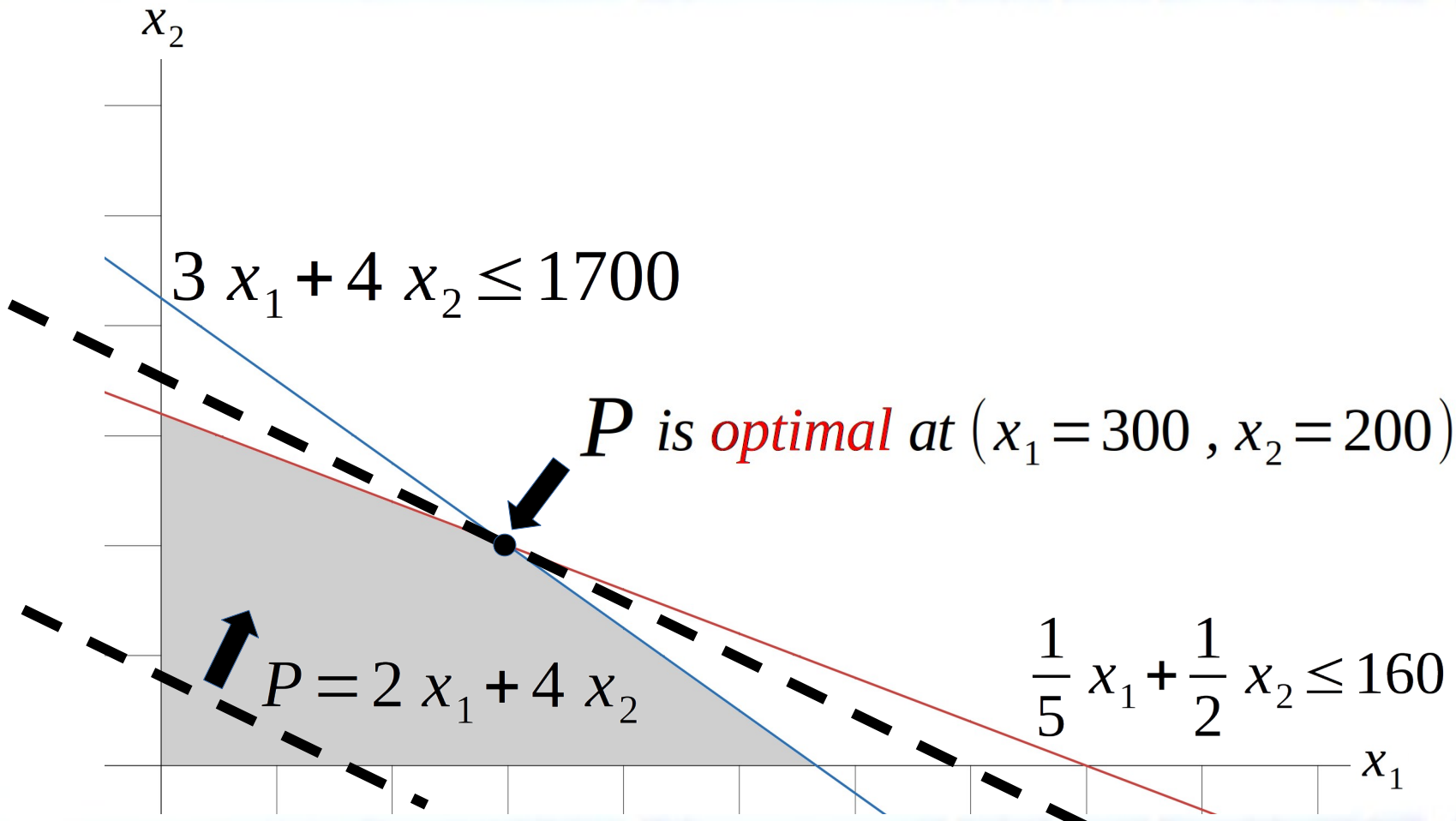
$$\frac{1}{5}\,x_1 + \frac{1}{2}\,x_2 \leq 160$$

"Constraints"

Modeling an LP Problem



Modeling an LP Problem



Solving an LP Problem

- Graphical Method (aforementioned example)
- Simplex Method
- Interior Point Methods
- Branch-and-Bound Method
- Primal-Dual Hybrid Gradient
- and more…

LP-Related Domains

- Integer Programming (IP)
- Binary Integer Programming (BIP)
- Mixed-Integer Programming (MIP)
- Mixed-Integer Non-Linear Programming (MINLP)
- [Non]Convex Quadratic Programming (QP)
- Fractional Linear Programming (FLP)
- Second-Order Cone Programming (SOCP)
- and more…

Modeling an SCP as LP Problem

Modeling an SCP as an LP Problem

$$P = 2\,x_1 + 4\,x_2$$

"Objective Function"

$$3\,x_1 + 4\,x_2 \leq 1700$$

$$\frac{1}{5}\,x_1 + \frac{1}{2}\,x_2 \leq 160$$

"Constraints"

Modeling an SCP as an LP Problem

$$minimize \quad \sum_{j=1}^{n} c_j x_j$$

c_j = cost of set

$$s.t. \quad x_j \in (0,1) \qquad j = 1,\ldots,n$$

$$\sum_{j=1}^{n} a_{ij} x_j \geq 1 \qquad i = 1,\ldots,m$$

a_{ij} = incidence matrix

Modeling an SCP as an LP Problem

$$\sum_{j=1}^{n} a_{ij} x_j \geq 1 \qquad i = 1, \ldots, m$$

$$Incidence = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{matrix} \geq 1 \\ \geq 1 \\ \geq 1 \end{matrix}$$

a_{ij} = incidence matrix

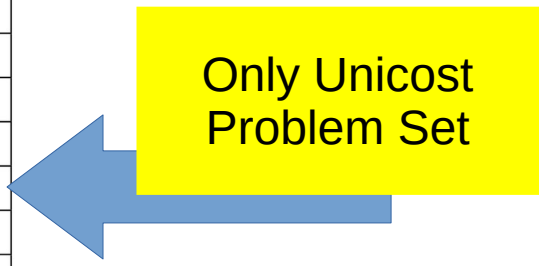
SCP Benchmarks

Representing an SCP Problem

- Balas, Ho, and Beasley
  - Published benchmark set in 1987
  - Expanded in 1990, 1997 across publications
  - Problem sets
    - “4 to 6”
    - “A – H”
- MIPLIB 2017

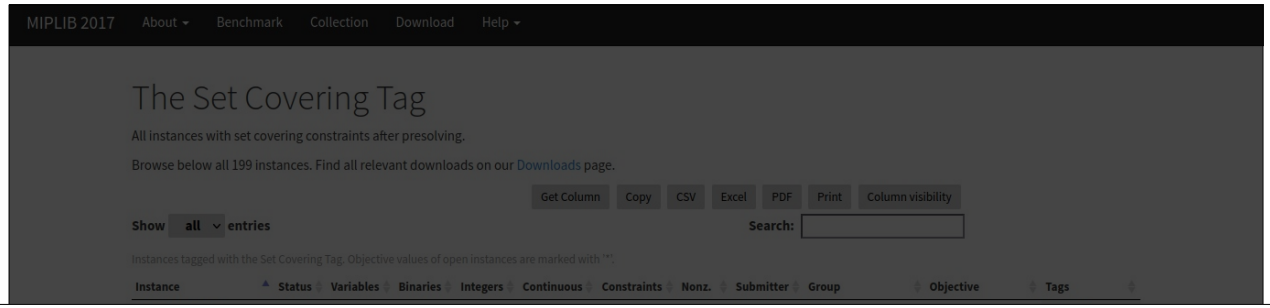
Beasley SCP Benchmark Compositions

Problem Set	Vertices	Sets	Density
4	200	1000	2%
5	200	2000	2%
6	200	1000	5%
A	300	3000	2%
B	300	3000	5%
C	400	4000	2%
D	400	4000	5%
E	50	500	20%
NRE	500	5000	10%
NRF	500	5000	20%
NRG	1000	10000	2%
NRH	1000	10000	5%



Problem Instance	Vertices	Sets	Density
Rail507	507	63009	1.3%
Rail516	516	47311	1.3%
Rail582	582	55515	1.2%
Rail2536	2536	1081841	0.4%
Rail2586	2586	92683	0.3%
Rail4284	4284	1092610	0.2%
Rail4872	4872	968672	0.2%

MIPLIB 2017

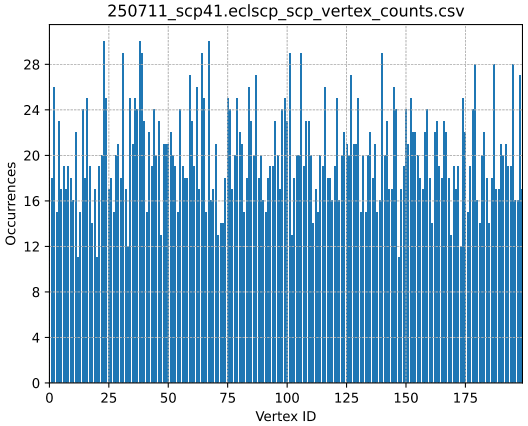


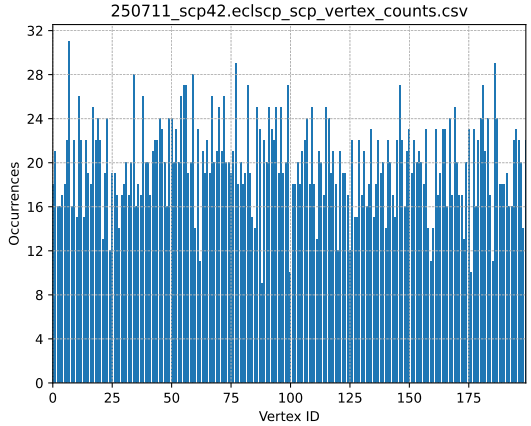
										knapsack
scpj4scip	open	99947	99947	0	0	1000	999893	Shunji Umetani	scp	128*	binary set_covering
scpk4	open	1e+05	1e+05	0	0	2000	1000000	Shunji Umetani	scp	318.0*	binary set_covering
scpl4	open	2e+05	2e+05	0	0	2000	2000000	Shunji Umetani	scp	259.0*	binary set_covering
scpm1	open	5e+05	5e+05	0	0	5000	6250000	Shunji Umetani	scp	540.0*	binary set_covering
scpn2	open	1e+06	1e+06	0	0	5000	12500000	Shunji Umetani	scp	485.0*	binary set_covering

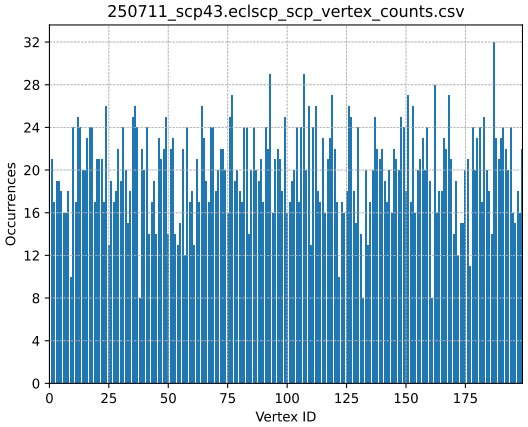
Some benchmarks are still “open” (optimal solution unknown)!

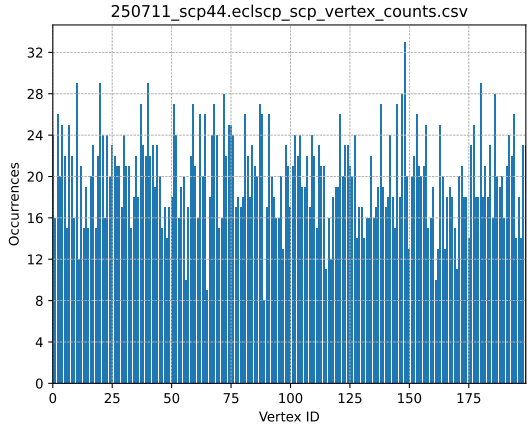
https://miplib.zib.de/

Beasley SCP Benchmark Compositions

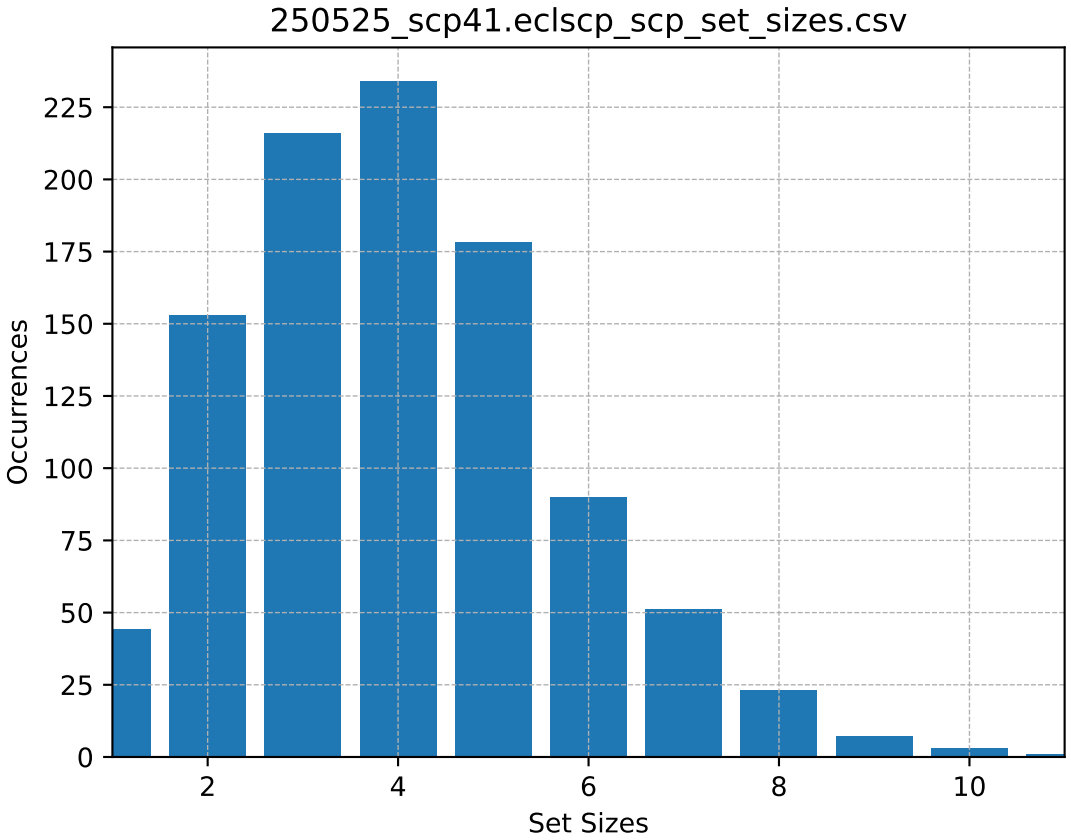








Beasley SCP Benchmark Compositions



Related Work

Closed-Source Implementations

- CPLEX (IBM)
- Gurobi
- Xpress (FICO)
- Mosek

Open-Source Implementations

- cuOpt (NVIDIA – GPU)
  - Uses Simplex Method
- CyLP / CBC (COIN-OR – CPU)
  - Uses Presolving
- HiGHS (ERGO-Code – CPU)
  - Uses Dual Simplex
- SCIP (SCIP-OPT – CPU)
  - Uses Primal / Dual

ECL-SCP Implementations

Experimental Methodology

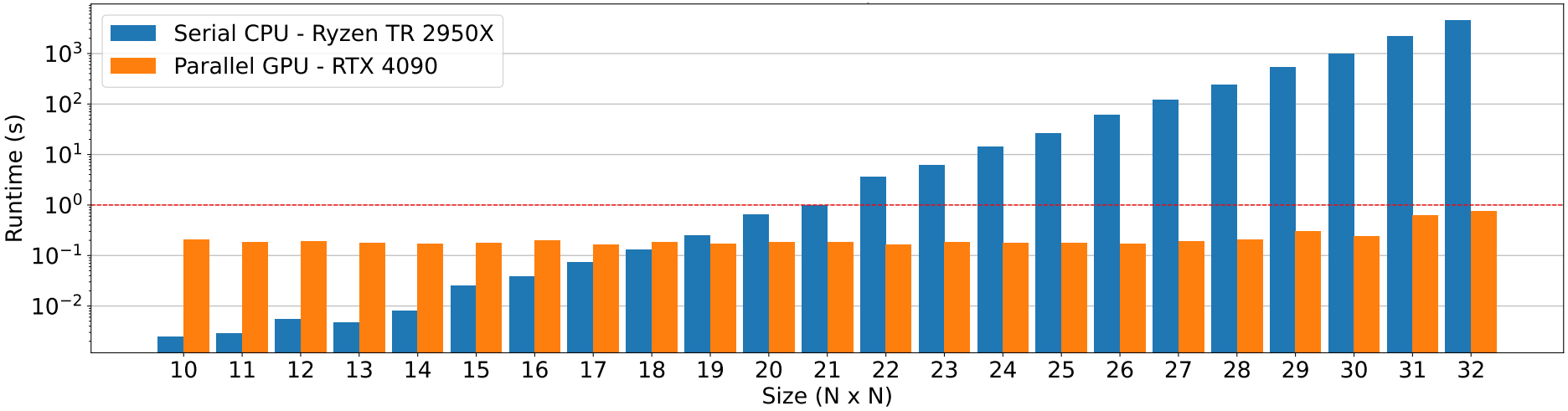
- AMD Ryzen Threadripper 2950X, 16 cores, 3.5GHz, gcc 13.3

	PE	SM	PE/SM	Mem (GB)	NVCC
RTX 4090	16384	128	128	24	12.6
RTX 3090	10496	82	128	24	12.0
A100	6912	108	64	40	12.0
GTX 1650	1024	16	64	4	12.6

ECL-SCP Brute-Force Implementation

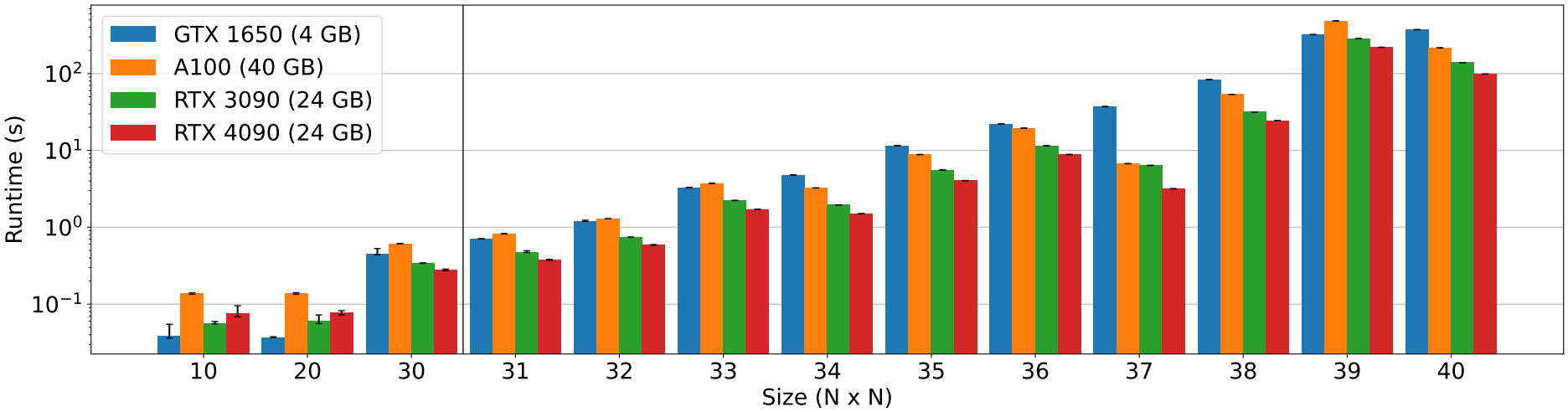
ECL-SCP Naïve CPU/GPU Brute Force

- Exhaustively iterate over all N-bit combinations
  - Runtime (s) vs. SCP Size - Ser-CPU to Par-GPU



ECL-SCP Naïve GPU Brute Force

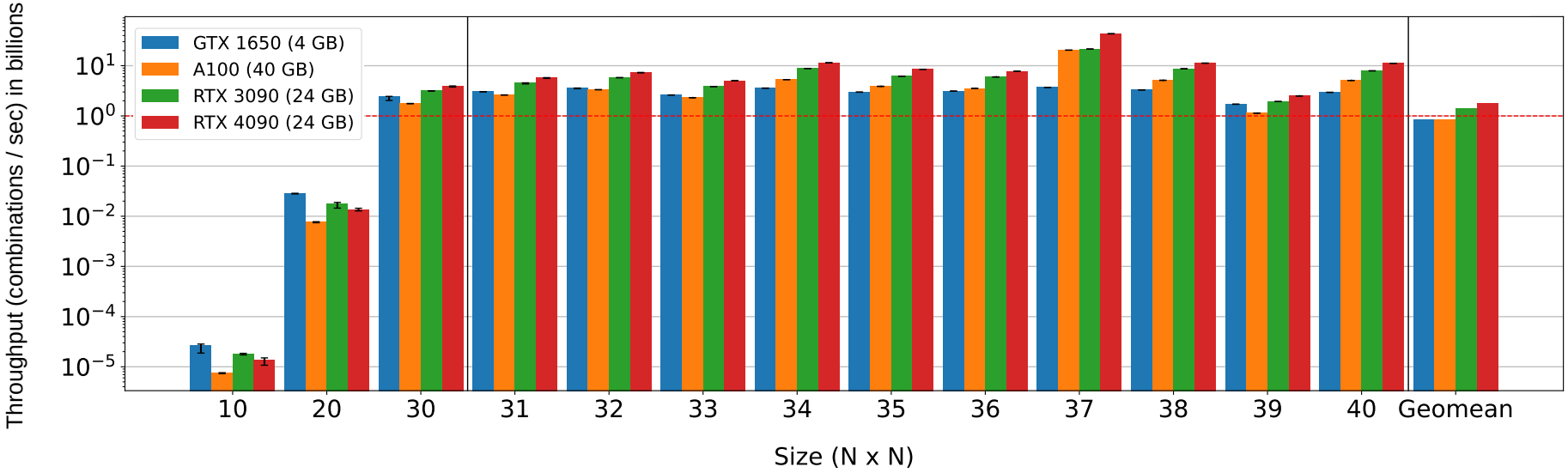
- With 40 bits, runtime is already > 100 sec!
  - Runtime (s) vs. SCP Size – Device Comparison



Only 10 more bits than CPU!

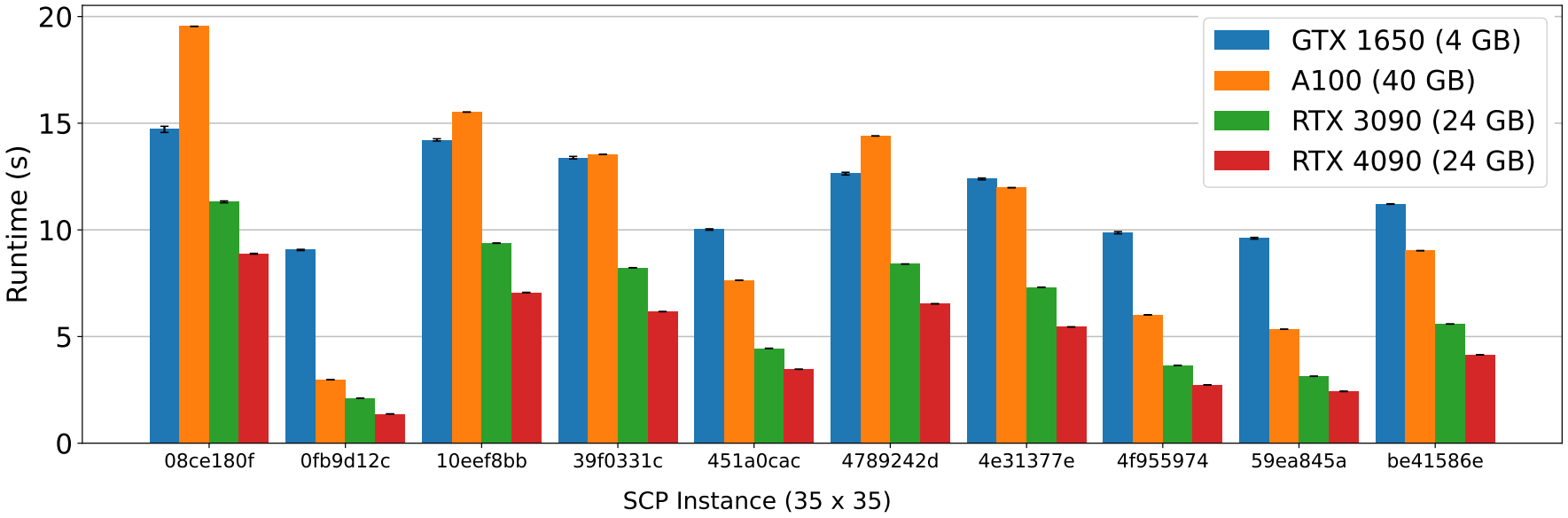
ECL-SCP Naïve GPU Brute Force

- GPUs become saturated after 30 bits
  - Throughput (combinations / s) in billions



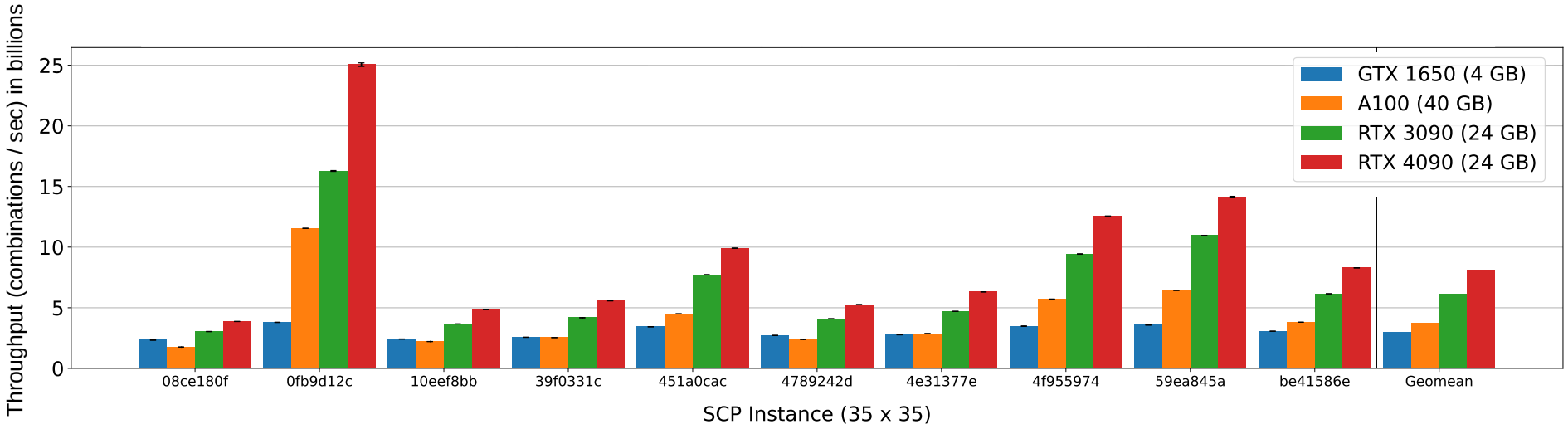
ECL-SCP Naïve GPU Brute Force

- Benchmark set compositions affect GPU runtimes
  - Runtime (s) vs. SCP (35 x 35) – Device Comparison



ECL-SCP Naïve GPU Brute Force

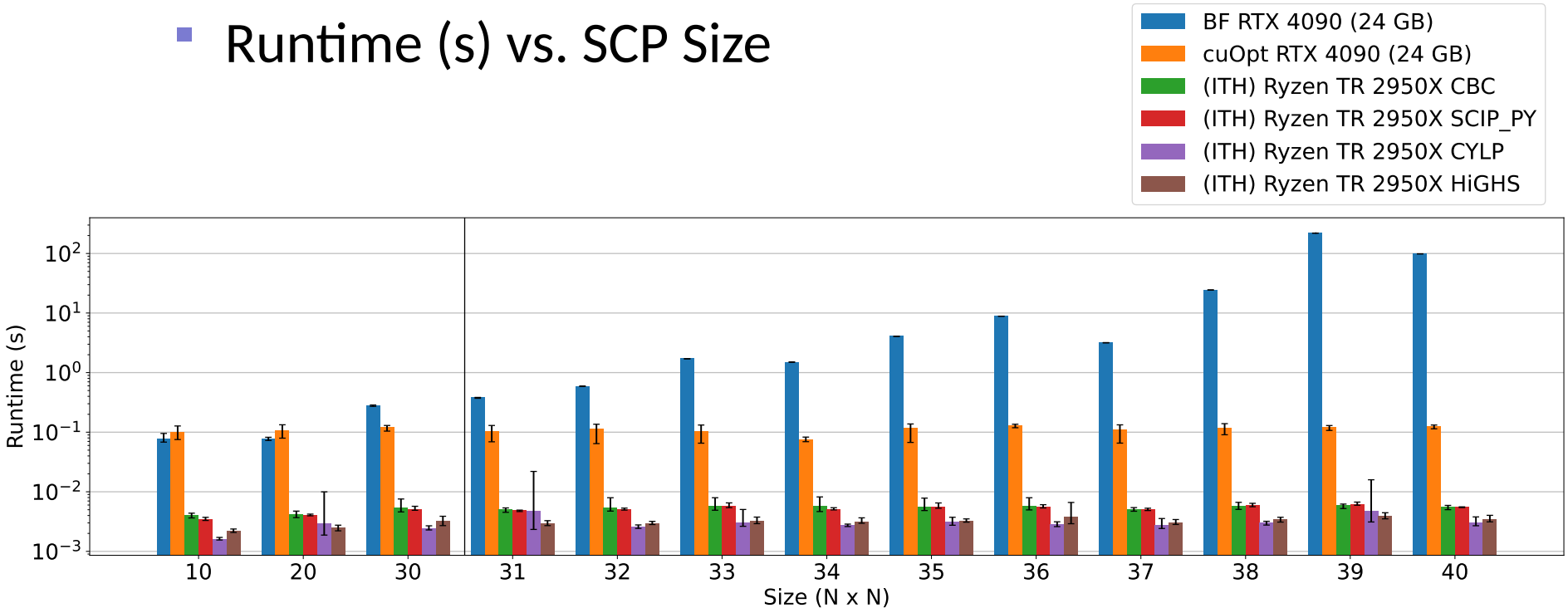
- Reached throughput of 25 billion 35-bit set combinations per second!
  - Throughput (combinations / s) in billions



Third-Party Comparisons

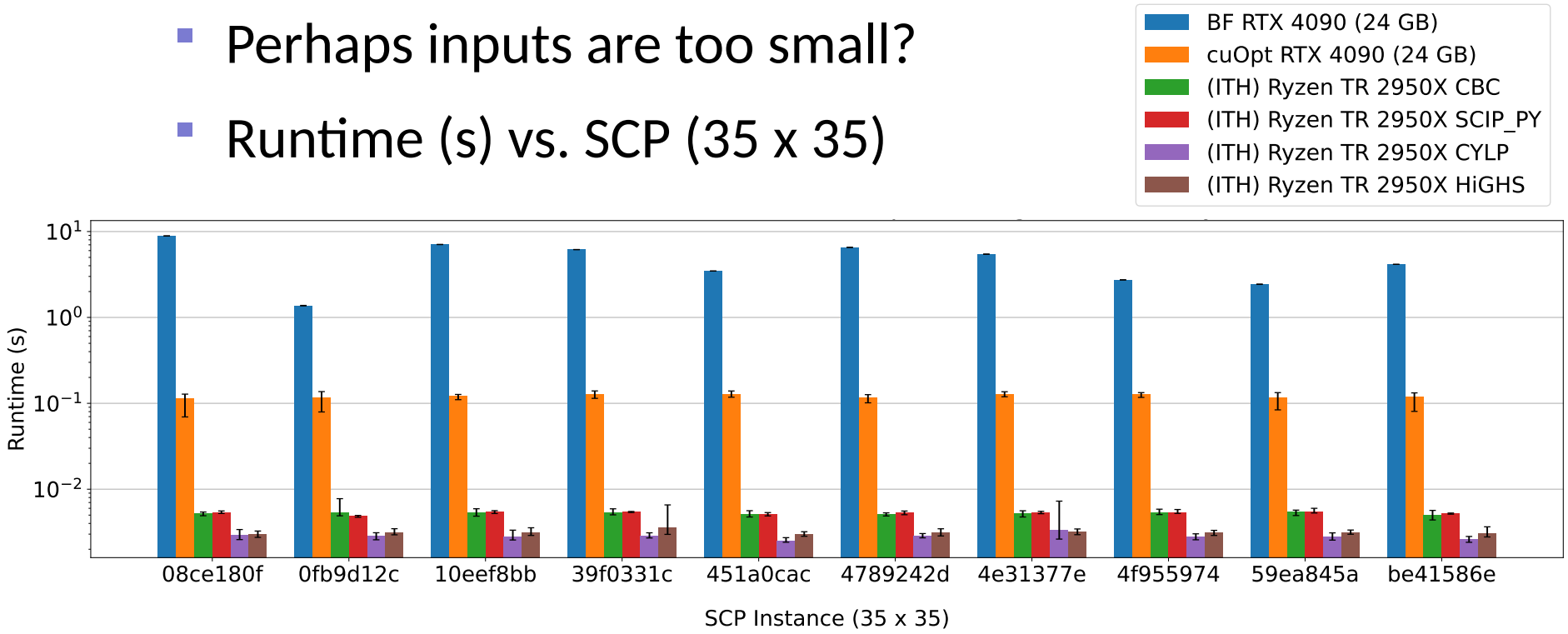
ECL-SCP BF Against 3rd Party

- Brute Force is not the best approach
  - cuOpt is also order of magnitude slower
  - Runtime (s) vs. SCP Size



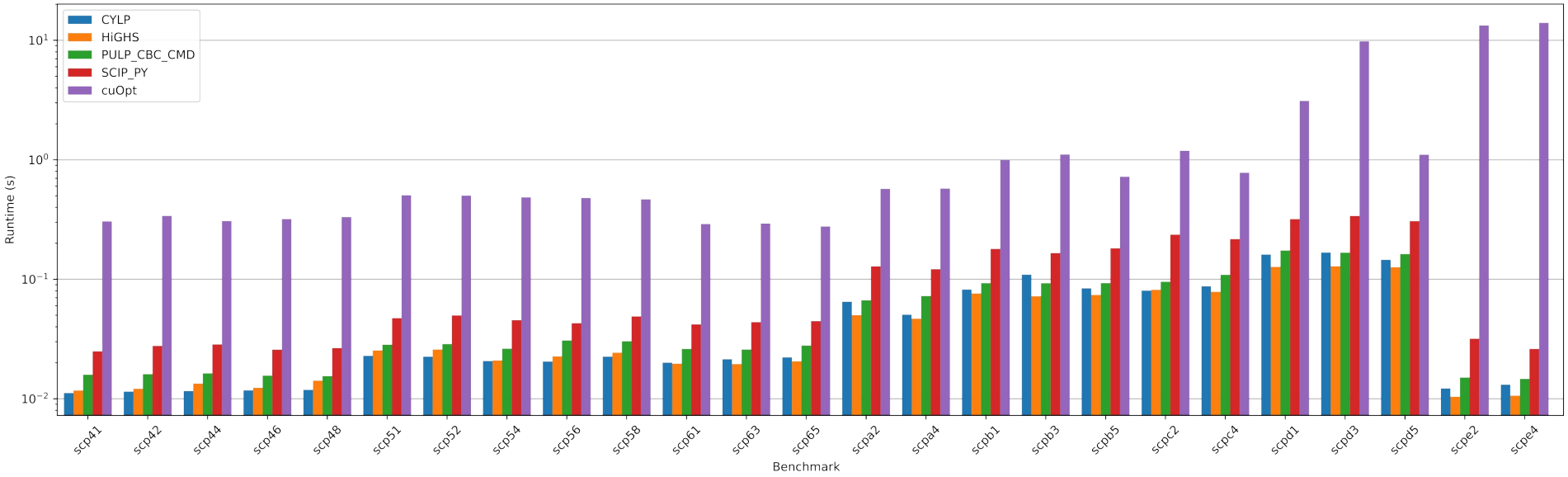
ECL-SCP BF Against 3rd Party

- Less variation in runtime against set composition
  - Perhaps inputs are too small?
  - Runtime (s) vs. SCP (35 x 35)



3rd Party on Beasley SCPs

- Open-source solvers take the lead on small SCPs
  - Runtime (s) vs. Beasley SCP



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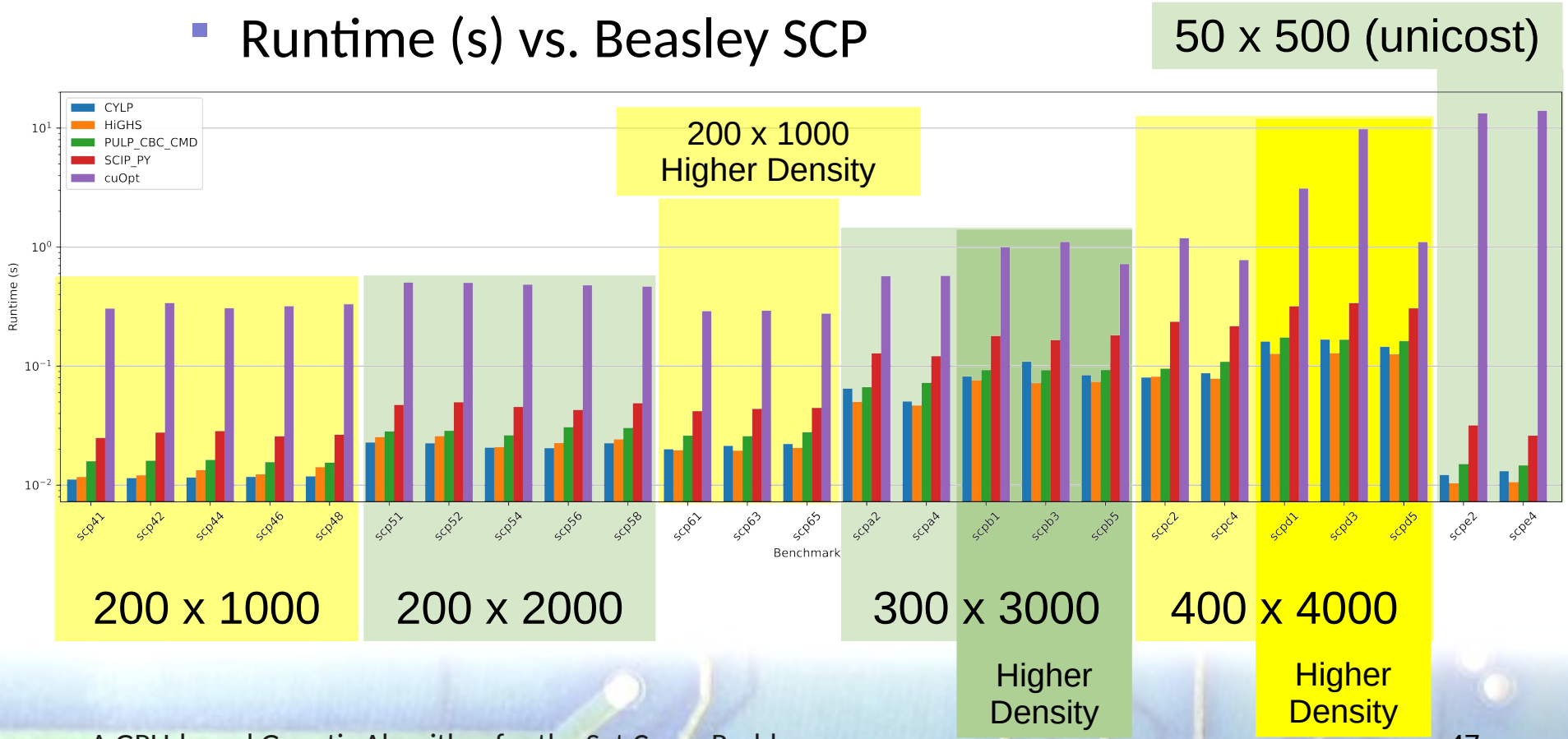
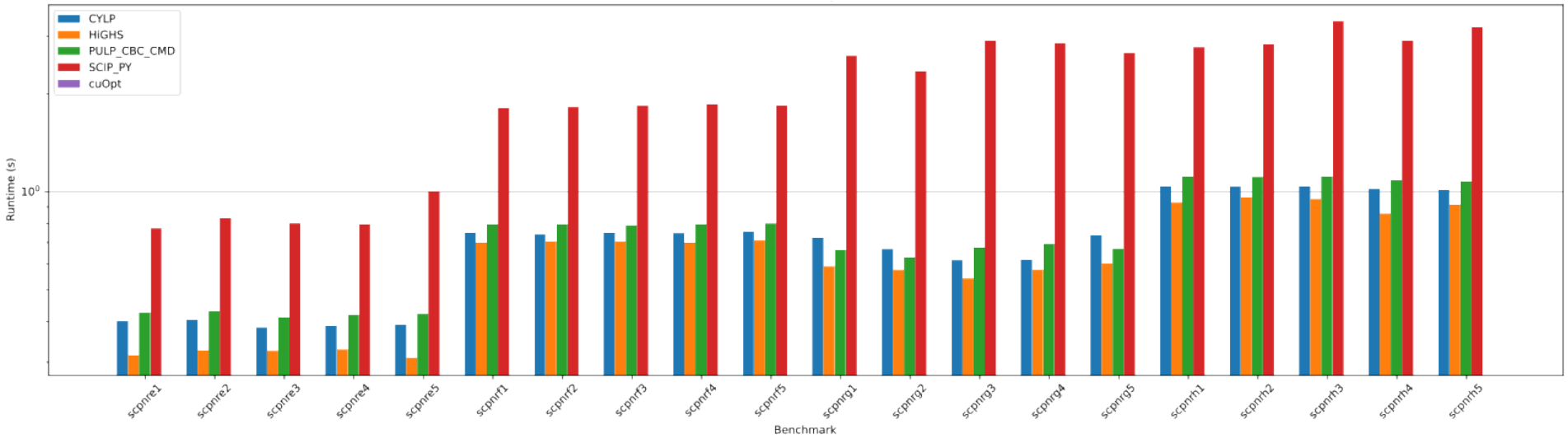


3rd Party on Beasley SCPs

- Open-source solvers take the lead on small SCPs
  - cuOpt times out for all of these inputs at 5 minutes
  - Runtime (s) vs. Beasley SCP



3rd Party on Beasley SCPs

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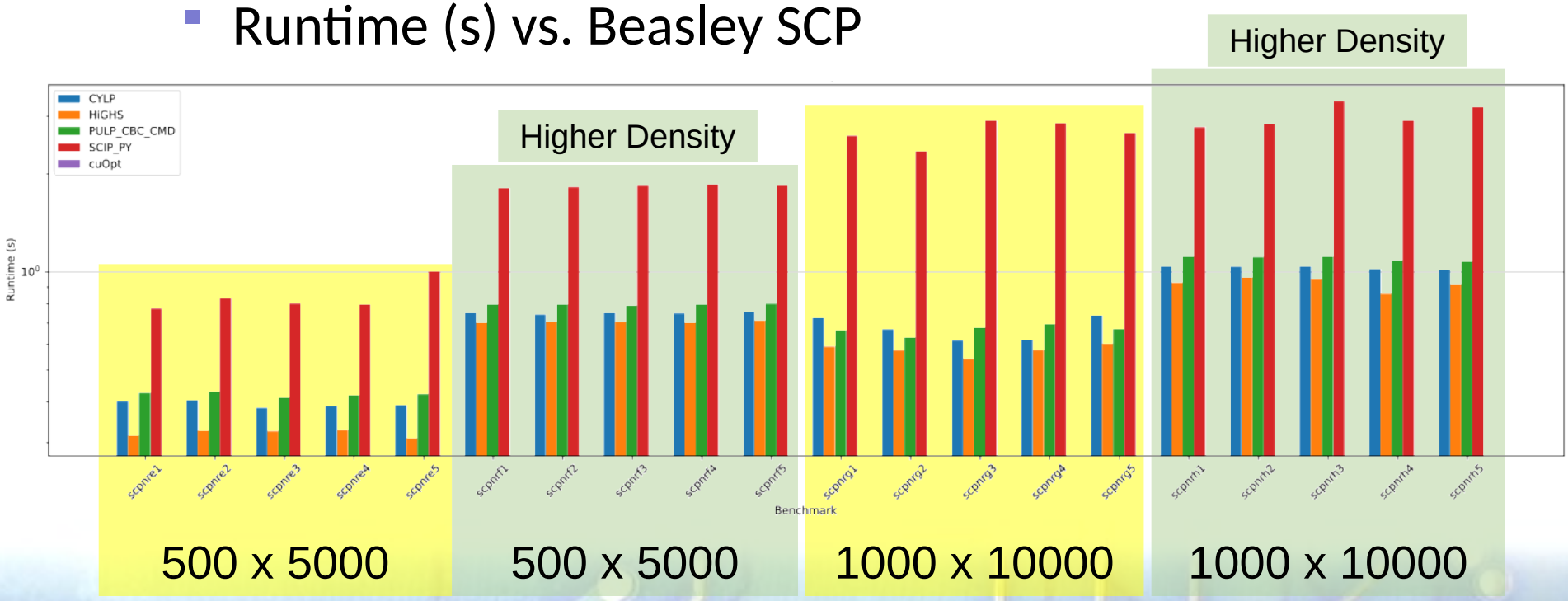
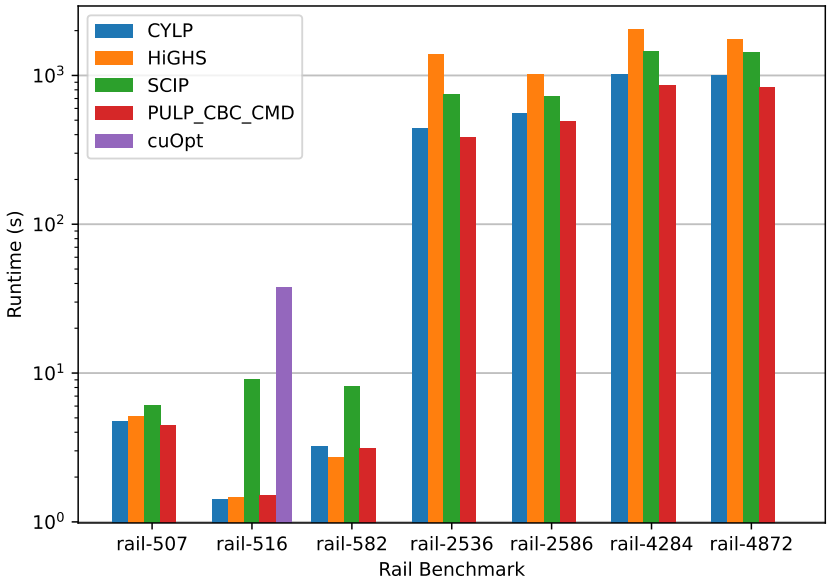
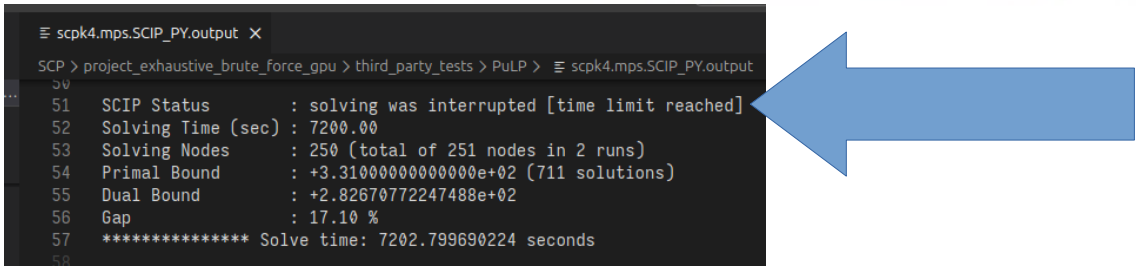


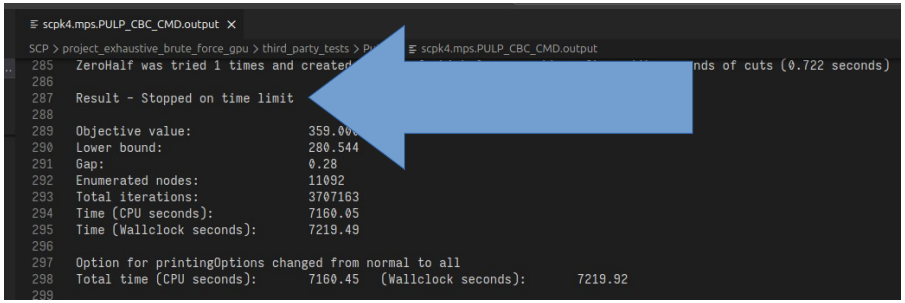
3rd Party on Beasley SCPs

- Rail benchmarks have room for improvement
  - cuOpt times out for all but one input at 2 hours
  - Runtime (s) vs. Beasley SCP – Rail benchmarks

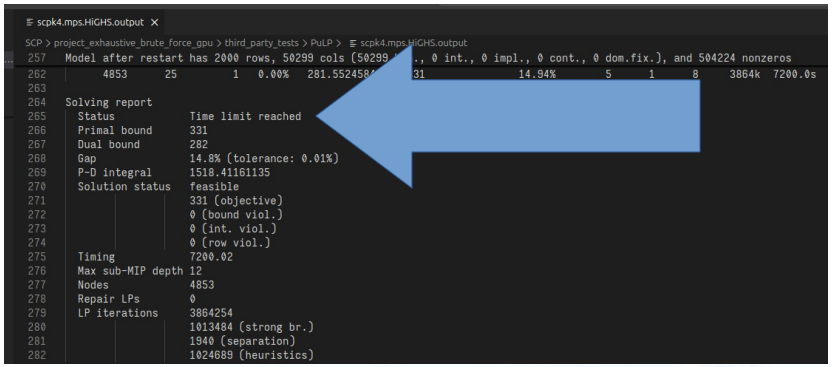


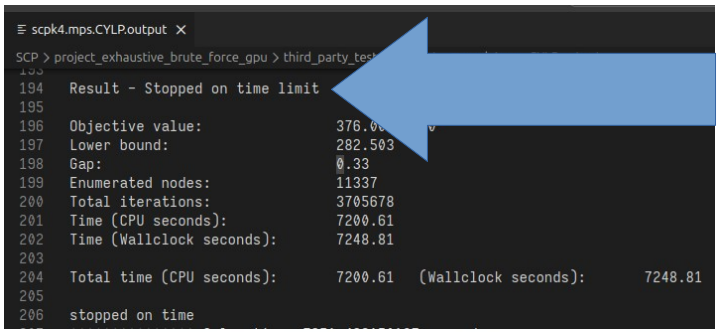
3rd Party Codes with Large Inputs





Large inputs (2,000 x 100,000) time out at 2 hours





ECL-SCP
Maximal Independent Set ("MIS")
and
Genetic Algorithm ("GA")
Parallel GPU Implementation

ECL-SCP Phase I - MIS

- First, find an MIS for vertices with respect to their "chosen" cheapest set
- If all vertices agree on that set, that set gets added to the running solution
  - The same applies to all other "agreed sets"
- Repeat for all vertices that remain, excluding already accounted-for vertices

ECL-SCP Phase II – GA

- Second, feed the solution obtained from the MIS phase into the GA
- Simulate generations of the population of organisms, taking into account organism fitness
  - Fitness = Feasibility + (1 / Cost)
- Organisms with higher fitness are more likely to generate offspring in the following generation

ECL-SCP Phase II – GA

- The most-fit organism always survives

- Creating the next generation of organisms (the new population) consists of
  - Generating new offspring (“crossover op”)
  - Mutating the alleles (“mutation op”)
  - Minimization
  - Ensuring feasibility

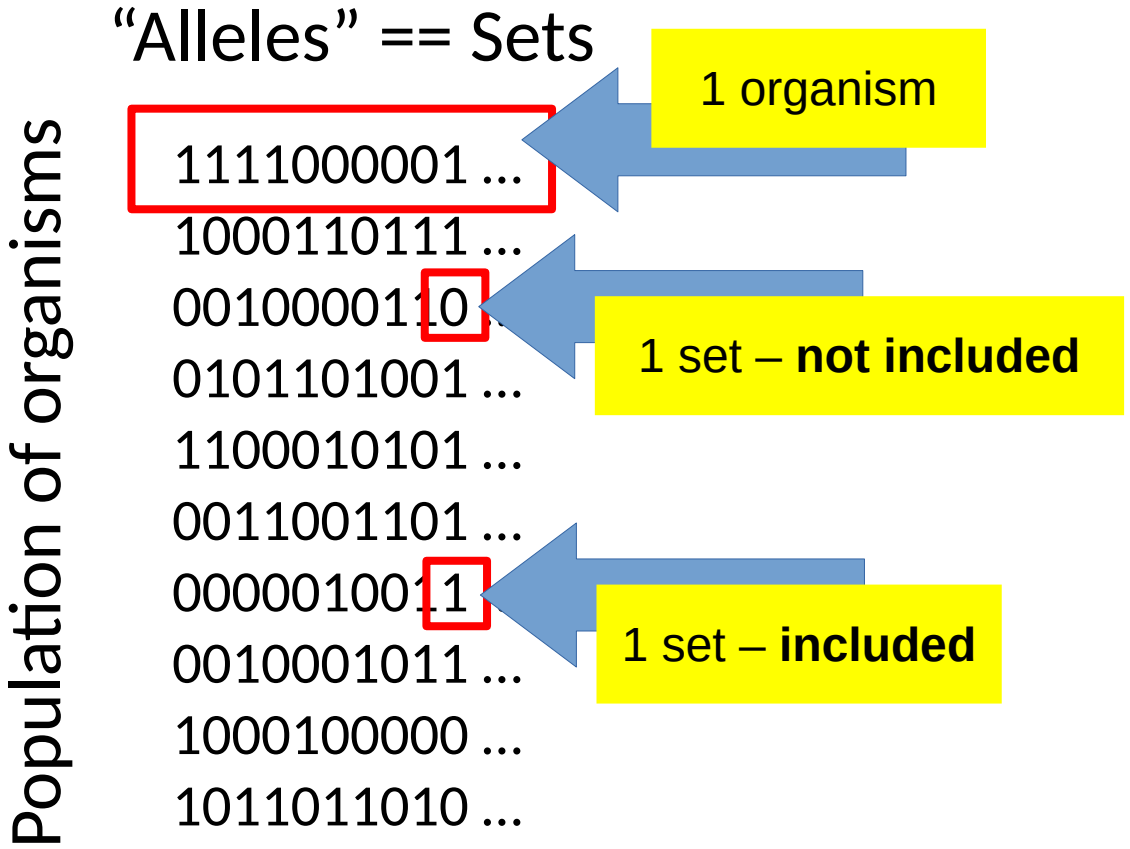
ECL-SCP Phase II – GA Population

"Alleles" == Sets

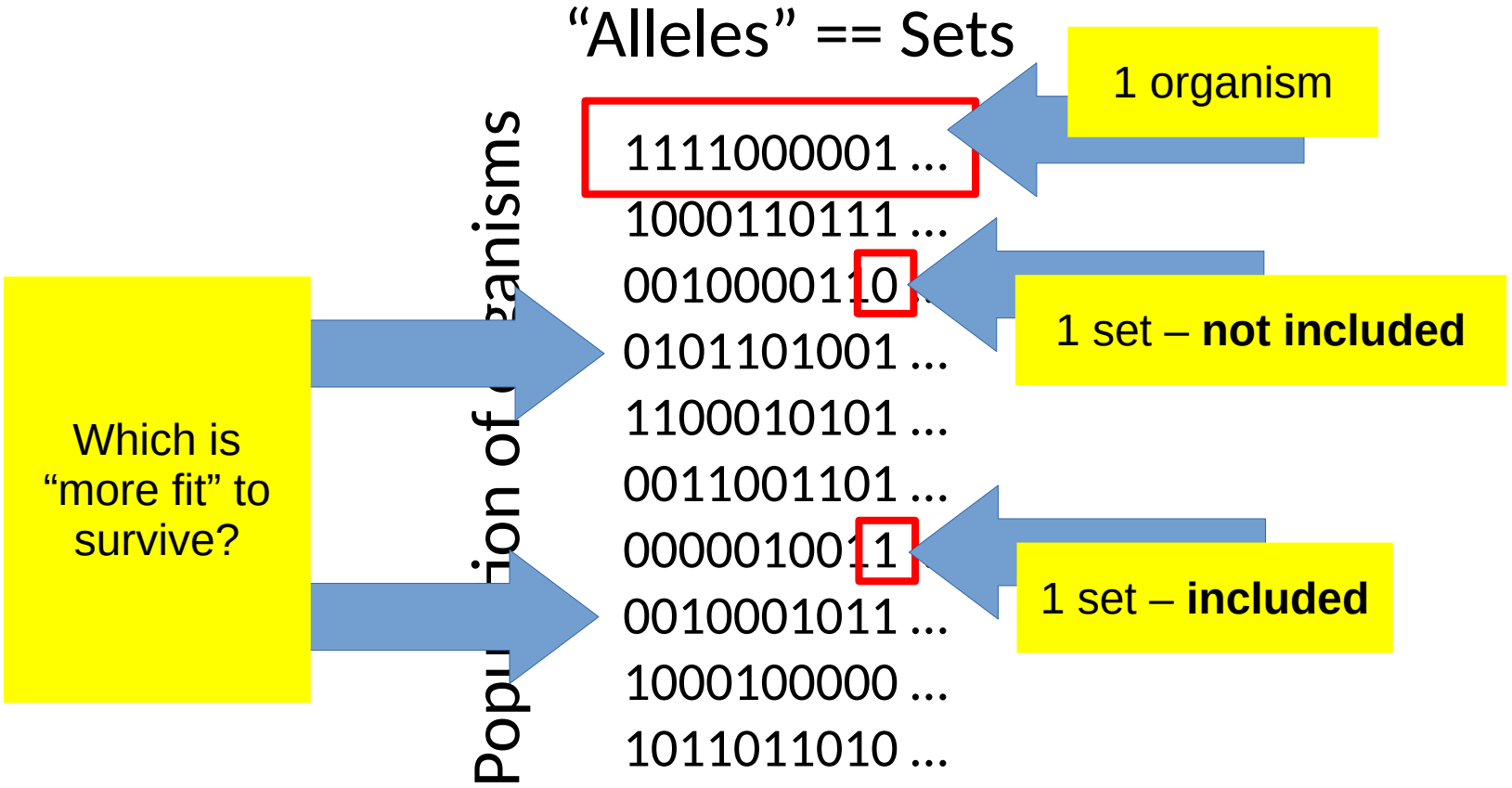
Population of organisms

1111000001 …
1000110111 …
0010000110 …
0101101001 …
1100010101 …
0011001101 …
0000010011 …
0010001011 …
1000100000 …
1011011010 …

ECL-SCP Phase II – GA Population



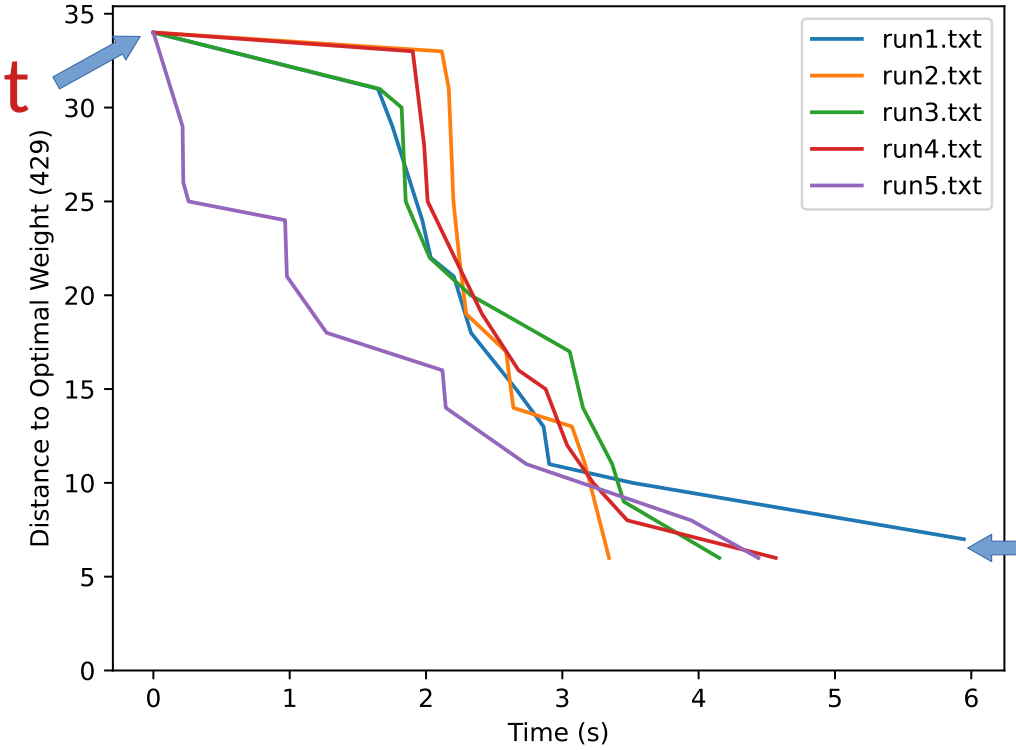
ECL-SCP Phase II – GA Population



ECL-SCP MIS + GA on (200 x 1000)

- Distance to Optimal vs. Time - SCP41

MIS Result

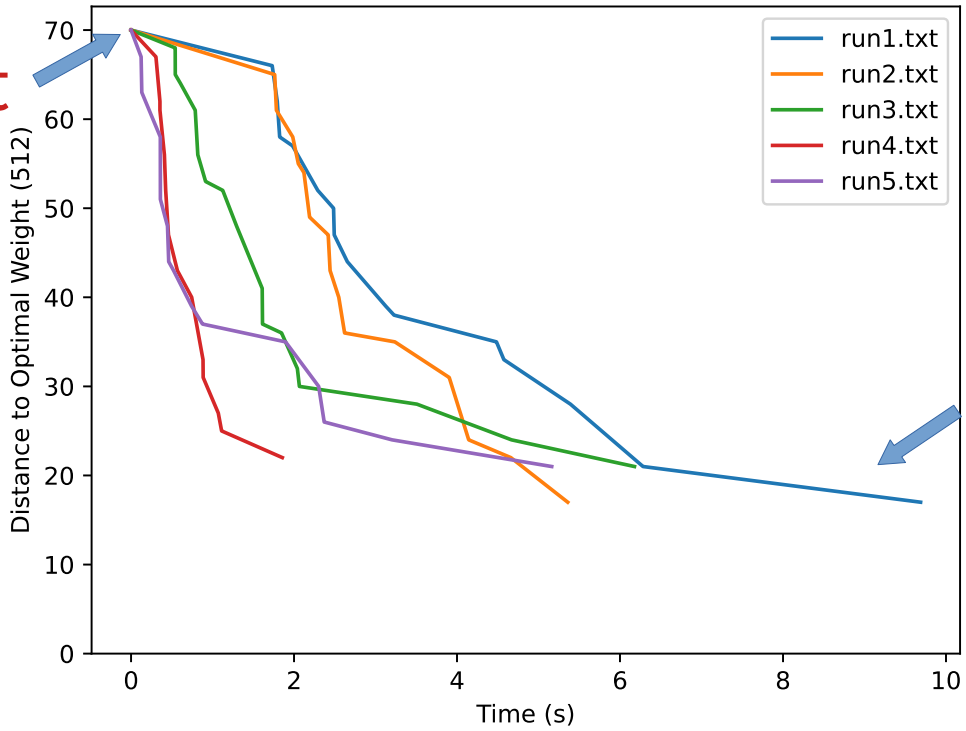


GA Results

ECL-SCP MIS + GA on (200 x 1000)

- Distance to Optimal vs. Time - SCP42

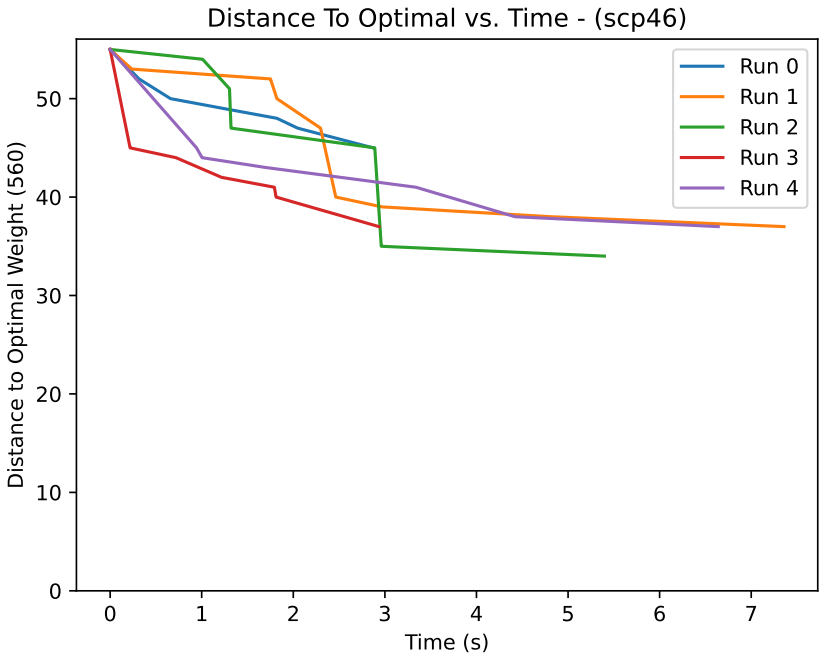
MIS Result



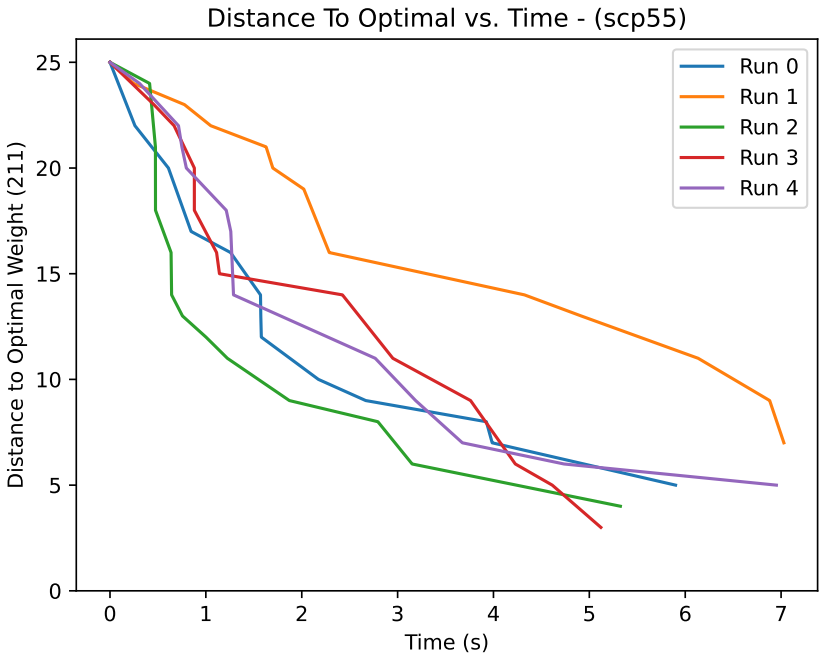
GA Results

ECL-SCP MIS + GA

(200 x 1000)

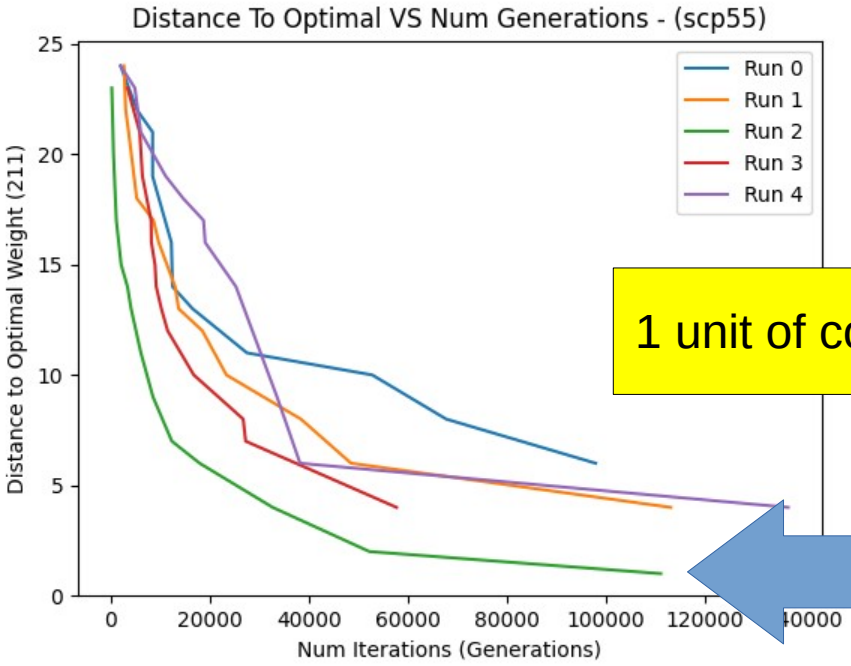


(200 x 2000)



ECL-SCP MIS + GA

(200 x 2000)



1 unit of cost from optimal!!

ECL-SCP MIS Rail Results

Problem Instance	Vertices	Sets	Optimal	Factor
Rail507	507	63009	172.14	1.24x
Rail516	516	47311	182.00	1.10x
Rail582	582	55515	209.71	1.19x
Rail2536	2536	1081841	688.39	1.28x
Rail2586	2586	92683	935.92	1.24x
Rail4284	4284	1092610	1054.05	1.31x
Rail4872	4872	968672	1509.63	1.26x

Future Work

- Take into account how valuable a particular allele may be with respect to the population
  - "The more-fit organisms tend to have this set as part of their genes"
- Optimize solution before feeding into Phase II GA (e.g., removing redundant sets)
- Process each connected-component as a separate problem and subsequently combine

Future Work

- Further comparisons against 3rd-party codes
- Optimize implementations
  - Can establish lower and upper bounds
- Tune GA configurations
- Looking to apply to problems with more than 5,000 vertices and 1,000,000 sets!

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Summary

- Brute-force approaches become untenable even with the smallest of inputs (40 vertices x 40 sets)
- We present ECL-SCP, a parallel GPU Maximal Independent Set + Genetic Algorithm SCP implementation
- Within 1 unit of cost of the optimal within seconds and match 97.7% chosen sets with solver output!

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ECL
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