# Trees & Heaps

Week 12

Gaddis: 20
Weiss: 21.1-3

CS 5301
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# Tree:
## non-recursive definition

- **Tree**: set of nodes and *directed* edges
  - **root**: one node is distinguished as the root
  - Every node (except root) has exactly exactly one edge coming into it.
  - Every node can have any number of edges going out of it (zero or more).
- **Parent**: source node of directed edge
- **Child**: terminal node of directed edge
- **Binary Tree**: a tree in which no node can have more than two children.



# Tree Traversals: examples

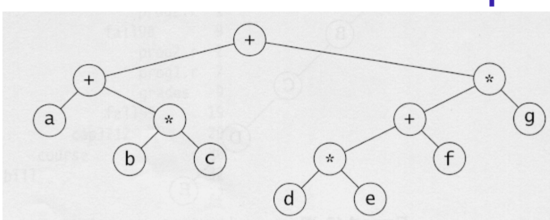



**Figure 4.14** Expression tree for (a + b * c) + ((d * e + f ) * g)

- **Preorder:** print node value, process left tree, then right `+ + a * b c * + * d e f g`
- **Postorder:** process left tree, then right, then print node value `a b c * + d e * f + g * +`
- **Inorder:** process left tree, print node value, then process right tree `a + b * c + d * e + f * g`



# Binary Trees: implementation

- Structure with a data value, and a pointer to the left subtree and another to the right subtree.

```
struct TreeNode {
  int value;        // the data
  TreeNode *left;   // left subtree
  TreeNode *right;  // right subtree
};
```

- Like a linked list, but two “next” pointers.
- There is also a special pointer to the root node of the tree (like head for a list).

```
TreeNode *root;
```

# Binary **Search** Trees

- A special kind of binary tree, used for efficient searching, insertion, and deletion.
- Binary Search Tree property:
  For every node X in the tree:
  - All the values in the **left** subtree are **smaller** than the value at X.
  - All the values in the **right** subtree are **larger** than the value at X.
- Not all binary trees are binary search trees
- An inorder traversal of a BST shows the values in sorted order

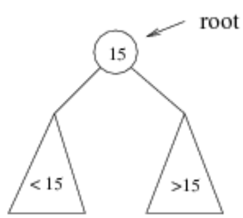


# Binary Search Trees: operations

- insert(x)
- remove(x) (or delete)
- isEmpty() (returns bool)
  - if the root is NULL
- find(x) (or search, returns bool)
- findMin() (returns <type>)
  - Smallest element is found by always taking the left branch.
- findMax() (returns <type>)
  - Largest element is found by always taking the right branch.



# BST: find(x)




Algorithm:

- if we are searching for 15 we are done.
- If we are searching for a key < 15, then we should search in the left subtree.
- If we are searching for a key > 15, then we should search in the right subtree.



# BST: find(x)

- Defined iteratively:

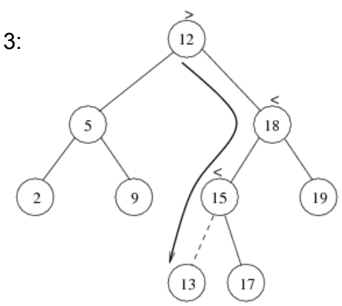
```
bool IntBinaryTree::searchNode(int num)
{
    TreeNode *p = root;

    while (p)
    {
        if (p->value == num)
            return true;
        else if (num < p->value)
            p = p->left;
        else
            p = p->right;
    }
    return false;
}
```

- Can also be defined recursively

# BST: insert(x)

- Algorithm is similar to find(x)
- If x is found, do nothing (no duplicates in tree)
- If x is not found, add a new node with x in place of the last empty subtree that was searched.

Inserting 13:






# BST: insert(x)

- Recursive function
- root is passed by reference to this function

```
void IntBinaryTree::insert(TreeNode *&nodePtr, int num)
{
    if (nodePtr == NULL) {
        // Create a new node and store num in it,
        // making nodePtr point to it
        nodePtr = new TreeNode;
        nodePtr->value = num;
        nodePtr->left = nodePtr->right = NULL;
    }
    else if (num < nodePtr->value)
        insert(nodePtr->left, num);     // Search the left branch
    else if (num > nodePtr->value)
        insert(nodePtr->right, num);    // Search the right branch
    // else nodePt->value == num, do nothing, no duplicates
}
```



# BST: remove(x)

- Algorithm is starts with finding(x)
- If x is not found, do nothing
- If x is found, remove node carefully.
  - Must remain a binary search tree (smallers on left, biggers on right).
  - The algorithm is described here in the lecture, the code is in the book (and on class website in **BinaryTree.zip**)



# BST: remove(x)

- Case 1: Node is a leaf
  - Can be removed without violating BST property
- Case 2: Node has one child
  - Make parent pointer bypass the Node and point to that child

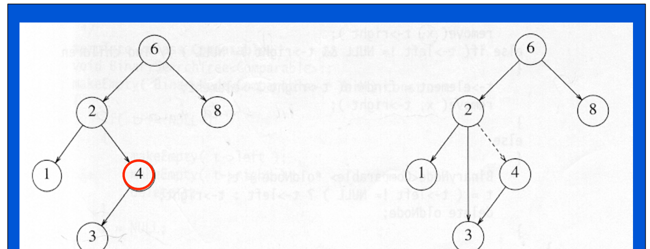



**Figure 4.24** Deletion of a node (4) with one child, before and after

Does not matter if the child is the left or right child of deleted node

# BST: remove(x)

- Case 3: Node has 2 children
  - Find minimum node in right subtree
    --cannot have left subtree, or it’s not the minimum
  - Move original node’s left subtree to be the left subtree of this node.
  - Make original node’s parent pointer bypass the original node and point to right subtree

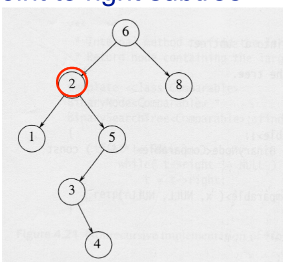
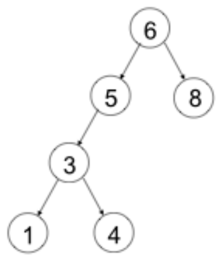



Figure 4.25 Deletion of a node (2) with two children, before and after




find minimum of 2’s right subtree (3) and move 2’s left subtree (1) to be it’s new left. Remove 2 (make 6 point to 5).



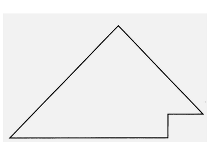
# Binary heap data structure

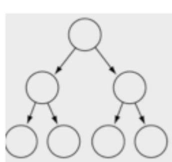
- A binary heap is a special kind of binary tree
  - has a restricted structure (must be complete)
  - has an ordering property (parent value is smaller than child values)
  - NOT a Binary Search Tree!
- Used in the following applications
  - Priority queue implementation: supports enqueue and deleteMin operations in O(log N)
  - Heap sort: another O(N log N) sorting algorithm.

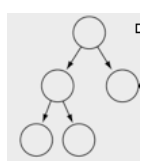


# Binary Heap:
## structure property

- **Complete binary tree**: a tree that is completely filled
  - every level except the last is completely filled.
  - the bottom level is filled left to right (the leaves are as far left as possible).



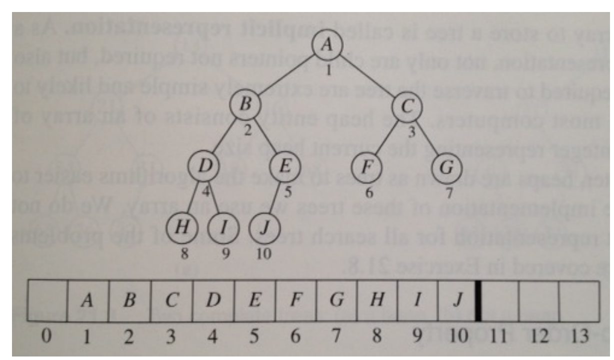






# Complete Binary Trees

- A complete binary tree can be easily stored in an array
  - place the root in position 1 (for convenience)



# Complete Binary Trees
## Properties

- In the array representation:
  - put root at location 1
  - use an int variable (size) to store number of nodes
  - for a node at position i:
    - left child at position `2i` (if 2i <= size, else i is leaf)
    - right child at position `2i+1` (if 2i+1 <= size, else i is leaf)
    - parent is in position `floor(i/2)` (or use integer division)
- There is a heap implementation on the class website in **Heap.zip**



# Binary Heap:
## ordering property

- In a heap, if X is a parent of Y, value(X) is less than or equal to value(Y).
  - the minimum value of the heap is always at the root.

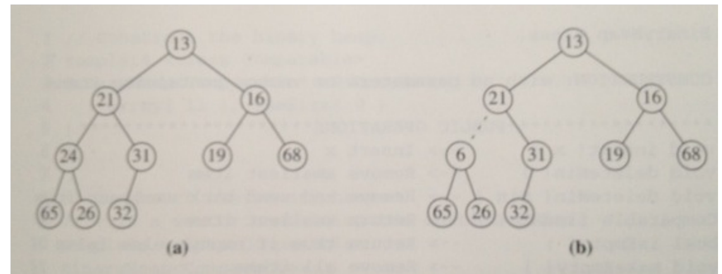



**Figure 21.3** Two complete trees: (a) a heap; (b) not a heap.



# Heap: insert(x)

- First: add a node to tree.
  - must be placed at next available location, size+1, in order to maintain a complete tree.
- Next: maintain the ordering property:
  - if x doesn't have a parent: done
  - if x is greater than its parent: done
  - else swap with parent, repeat
- Called “percolate up” or “reheap up”
- preserves ordering property



# Heap: insert(x)

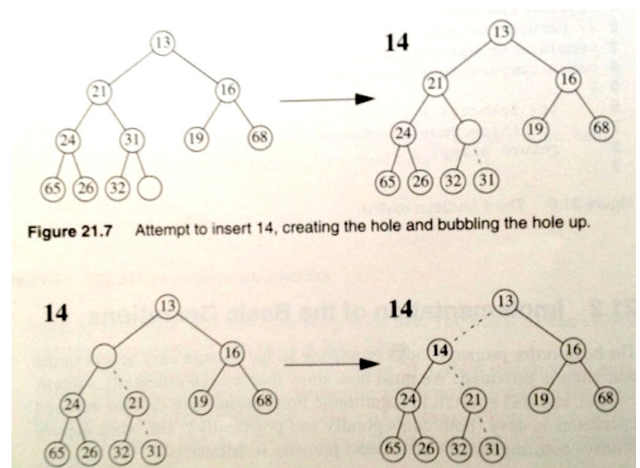



**Figure 21.7** Attempt to insert 14, creating the hole and bubbling the hole up.

**Figure 21.8** The remaining two steps required to insert 14 in the original heap shown in Figure 21.7.

# Heap: deleteMin()

- Minimum is at the root, removing it leaves a hole.
  - The last element in the tree must be relocated.
- First: move last element up to the root
- Next: maintain the ordering property, start with root:
  - if no children, do nothing.
  - if one child, swap with parent if it's smaller than the parent.
  - if both children are greater than the parent: done
  - otherwise, swap the smaller of the two children with the parent, and repeat on that child.
- Called "percolate down" or "reheap down"
- preserves ordering property



# Heap: deleteMin()

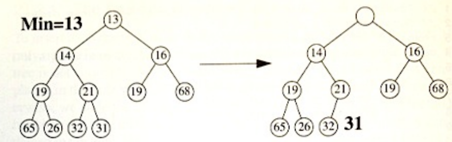



**Figure 21.10** Creation of the hole at the root.

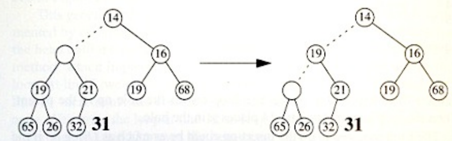



**Figure 21.11** The next two steps in the `deleteMin` operation.

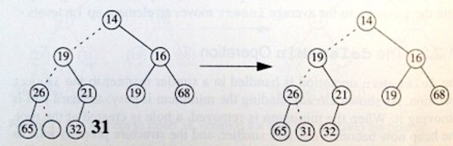



**Figure 21.12** The Last two steps in the `deleteMin` operation.