Reserving Processors by Precise Scheduling of Mixed-Criticality Tasks

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***Abstract*—Mixed-criticality (MC) scheduling has been proposed to mitigate the pessimism in real-time schedulability analysis that must provide guarantees for the worst case. In most existing work on MC scheduling, low-critical tasks are either dropped or degraded at the criticality mode switch in order to preserve the temporal guarantees for high-critical tasks. Recently, a different direction, called precise MC scheduling, has been investigated. In precise MC scheduling, no low-critical task should be dropped or degraded; instead, the platform processing capacity is augmented at mode switch to accommodate the additional workload by high-critical tasks. In contrast to prior work on this topic with respect to varying processor speed, this work investigates the precise scheduling problem of MC tasks when the number of available processors may vary at the mode switch. To address this new problem, we propose two alternative algorithms by adapting virtual-deadline-based EDF and by fluid scheduling, respectively, and provide a sufficient schedulability test for each. We also conduct schedulability experiments with randomly generated task sets to demonstrate the effectiveness of the proposed algorithms and the benefits of the new scheduling model.**

***Index Terms*—mixed-criticality tasks, precise scheduling, reserving processors, virtual deadlines, fluid scheduling.**

I. Introduction

To mitigate the pessimism in real-time schedulability analysis that must provide guarantees for the worst case, mixed-criticality (MC) scheduling has been proposed, featured by modeling a *single* system parameter with *multiple* estimates. In particular, two estimates on the worst-case execution time (WCET) of a task are the most commonly studied in the literature, where two system modes are defined depending on which of the two estimates are respected.

A significant body of existing research on MC scheduling was conducted in the direction that low-critical tasks are sacrificed during execution, completely or partially, in the worst case in order to provide the real-time guarantees to high-critical tasks. However, Ernst and Di Natale suggested that dropping or degrading service, even if for low-critical (rather than non-critical) tasks, may be infeasible or problematic for some applications [10]. Thus, the precise MC scheduling has been proposed [6], where even low-critical tasks are guaranteed full service. In precise MC scheduling, the system mode switch concerns degrading the active processors in typical scenarios for energy conservation while enabling the full capability of the underlying platform to preserve the real-time guarantees in the worst-case scenarios.

Energy efficiency is essential, especially for embedded systems, which often rely on unreliable energy sources such as batteries or energy harvesting devices. To improve energy efficiency, many modern processors are equipped with a feature, called *dynamic voltage and frequency scaling* (DVFS), which enables dynamic adjustment of processor voltage and frequency, *i.e.*, the speed of processors may vary during runtime. However, DVFS has a major limitation: it is not effective in reducing static/leakage power consumption, which may elevate to 50% or more of the overall power consumption [12]. By contrast, *dynamic power management* (DPM) and deep sleep modes can lead to significant energy conservation resulted from the power-down of a number of system components—not only the cores but also their associated caches, translation look-aside buffers, *etc.* Similar to DVFS yielding varying-speed processors, DPM and deep sleep modes may cause the number of active processors to vary as well.

In this paper, we consider a new precise MC scheduling problem inspired by DPM and deep sleep modes. We interpret the two WCET estimates of an MC task as a typical-case and a worst-case upper bounds on its execution time. While the worst-case bound shall never be exceeded, whether the actual execution time during runtime will respect the typical-case bound or not is unknown prior to runtime. If all actual execution times are indeed within their typical-case bounds, then the tasks are executed only on a subset of the processors, leaving other processors inactive and in the deep sleep mode by DPM. Once it is observed that any actual execution time exceeds its typical-case bound, all processors should be immediately activated to ensure that all deadlines are still met. As suggested by its name, the typical-case upper bound should be respected in most of the time and the worst-case mode should be a rare event. Therefore, the proposed precise scheduling of MC tasks could significantly reduce the system energy consumption while preserving the real-time guarantees for all tasks even in the worst case.

Related Work. Since it was introduced by Vestal [19], MC tasks and their scheduling have attracted a huge amount of interest in the real-time systems research community. (Please see [8] for a comprehensive survey on this topic.) Initially, most works were directed to scenarios where all low-critical tasks

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are completely dropped if any high-critical task behaves its worst case. More recently, this over-sacrificing was criticized, and gradual degradation of low-critical tasks was investigated. To provide degraded service, the imprecise MC model [7] was proposed, where the execution of low-critical tasks is reduced but not dropped even in the worst case. Several subsequent works [4, 7, 11, 13, 14, 17] explored various definitions of this execution reduction. To eliminate such reduction, the problem of precise MC scheduling was proposed and investigated on varying-speed uniprocessors [6, 20] and multiprocessors [18].

Contributions. In this paper, we extend the research on precise scheduling of MC tasks to another dimension. In contrast to prior work that focused on varying the speed of all processors simultaneously, we investigate an alternative approach where the number of active processors may vary in different modes. We formalize the system model and define an MCrp-schedulability problem. To address this problem, we propose two algorithms that are based on virtual deadlines and fluid scheduling, respectively, For each algorithm, we derive a sufficient schedulability test to validate the MCrp-schedulability of the system prior to runtime. To our knowledge, this is the first work on precise scheduling of MC tasks for reserving processors. Furthermore, our schedulability experiments demonstrate the merits of this work over prior related work on varying-speed processors.

Organization. In the rest of this paper, we introduce our system model and problem statement (Sec. II), present two new algorithms based on virtual deadlines (Sec. III) and fluid scheduling (Sec. IV), respectively, as well as their schedulability tests, evaluate the proposed model and algorithms (Sec. V), and conclude (Sec. VI).

II. System Model and Problem Statement

We consider the scheduling of a set of n implicit-deadline sporadic MC tasks $\tau = \{\tau_1, \tau_2, \cdots, \tau_n\}$. Each task τ_i is invoked recurrently with a minimum separation of T_i time units, where each invocation is called a *job* of τ_i and T_i is called the *period* of τ_i. We also restrict our attention to implicit deadlines. In other words, T_i is also the *relative deadline* for each task τ_i, and every job of τ_i has an absolute deadline T_i time units after its release. The worst-case execution time (WCET) of each task τ_i is estimated a two criticality levels: a low-criticality (L-) estimate C_i^L and a high-criticality (H-) estimate C_i^H, where it is assumed that $\forall i, 0 < C_i^L \leq C_i^H \leq T_i$. Furthermore, if $C_i^L = C_i^H$ for task τ_i so that τ_i *cannot* trigger a *mode switch* as to be described next, then we say τ_i is a LO-task; by contrast, if $C_i^L < C_i^H$ for task τ_i so that τ_i *could* trigger a *mode switch* as to be described next, then we say τ_i is a HI-task. We denote the set of LO-tasks (HI-tasks, respectively) by $\mathcal{T}_{\text{LO}}$ ($\mathcal{T}_{\text{HI}}$, respectively). We also refer to a job of a LO-task (HI-task, respectively) as LO-job (HI-job, respectively) for short.

Reserving processors and mode switch. We consider a multiprocessor platform consisting of M^H identical processors, each of which has a normalized speed 1.0. In the runtime, if the L-estimates are respected, *i.e.*, *all* jobs are finished within their L-WCETs, then we say the system is in *L-mode*; if the L-estimates are exceeded, *i.e.*, *some* jobs need to execute beyond their L-WCETs and up to their H-WCETs, then we say the system is in *H-mode*. Note that the H-estimates are assumed to be always respected. In other words, any job that has cumulatively executed for its H-WCET, *i.e.*, C_i^H, yet still not completed, is considered as *erroneous* and would be terminated. That is, only HI-tasks, for which $C_i^L < C_i^H$, could trigger a mode switch. The system begins with L-mode and the amount of execution completed for each job is being monitored during runtime. If any job has cumulatively executed for its L-WCET, *i.e.*, C_i^L, but still requires further execution, then the system is immediately notified and switched to H-mode. The system can recover to L-mode once *all* processors become idle. We require that only $M^L < M^H$ processors are used to actively execute tasks in τ in L-mode, while the remaining $M^\Delta = (M^H - M^L)$ processors are reserved. Nonetheless, once the system is switched to H-mode, all M^H processors are devoted to execute tasks in τ.

Note that, in contrast to the majority of existing works on MC scheduling, *no task is entirely or partially dropped* upon a mode switch, and every job must meet its absolute deadline in any system mode. The difference between the two WCET estimates upon mode switch, *i.e.*, $C_i^H - C_i^L$, is compensated by the additional M^Δ active processors.

In this paper, we assume that the preemption and migration overheads, *e.g.*, due to memory interference, are negligible. Or, equivalently, we assume these overheads are pessimistically taken into account in the WCET estimates.

We denote the utilization of a task τ_i in L- and H-modes, respectively, by

$$u_i^L = \frac{C_i^L}{T_i} \quad \text{and} \quad u_i^H = \frac{C_i^H}{T_i}.$$

Since $C_i^L = C_i^H$ holds for every LO-task, it also holds $u_i^L = u_i^H$ for such task. We further denote the total utilization of the set of LO-tasks and the set of HI-tasks in L- and H-modes, respectively, by

$$U_{\text{LO}} = \sum_{\tau_i \in \mathcal{T}_{\text{LO}}} u_i^L = \sum_{\tau_i \in \mathcal{T}_{\text{LO}}} u_i^H,$$

$$U_{\text{HI}}^L = \sum_{\tau_i \in \mathcal{T}_{\text{HI}}} u_i^L, \text{ and } U_{\text{HI}}^H = \sum_{\tau_i \in \mathcal{T}_{\text{HI}}} u_i^H.$$

We also define $\hat{u}_{\text{HI}}^L = \max_{\tau_i \in \mathcal{T}_{\text{HI}}} \{u_i^L\}$ and $\hat{u}_{\text{HI}}^H = \max_{\tau_i \in \mathcal{T}_{\text{HI}}} \{u_i^H\}$.

Problem Statement. We address the problem of scheduling the MC tasks on M^H unit-speed processors to meet all deadlines in *all* scenarios with the potential of reserving M^Δ processors, where $M^\Delta = M^H - M^L > 0$. We say the system is *MCrp-schedulable* if all deadlines are guaranteed to be met and the following constraints are respected.

- Tasks in τ only execute on M^L processors if *all* jobs finish within C_i^L time units;
- Tasks in τ may execute on all the M^H processors if a *any* job (of a HI-task) executes for more than C_i^L time units (yet finishes within C_i^H time units of execution).

III. Scheduling by Virtual Deadlines

In this section, we present a new algorithm to address the problem considered in this paper by leveraging an existing non-MC scheduler fpEDF and the MC scheduling technique of setting virtual deadlines.

A. *Algorithm* fpEDF *and* EDF-VD.

For scheduling ordinary *non-MC periodic* tasks with implicit deadlines, Baruah [1] developed fpEDF ("fp" stands for fixed-priority), which is also applicable to non-MC *sporadic* tasks with implicit deadline. Under fpEDF, *high-utilization* tasks for which the utilization exceeds 0.5 are statically prioritized, and the remaining tasks are scheduled according to global earliest-deadline-first (EDF). Assuming m identical unit-speed processors, a utilization-based sufficient schedulability test for fpEDF is as follows.

Theorem 1 (adapted from Theorem 4 in [1])**.** *Let U denote the total utilization of an implicit-deadline sporadic task set, in which the utilization of task τ_i is denoted by u_i. This task set is schedulable by* fpEDF *on m identical unit-speed processors if*

$$\forall i, u_i \leq 1.0 \quad \textit{and} \quad U \leq \frac{m+1}{2}.$$

On the other hand, EDF-VD ("VD" stands for virtual deadlines) was proposed to address the MC scheduling problem on a single processor and all LO-tasks are dropped upon the mode switch [2, 3]. Under EDF-VD, virtual deadlines are set to promote the execution of HI-tasks in L-mode and to leave slack for the potential extra workload at a mode switch from L to H. A common scaling factor (a constant less than 1.0) is used to determine the virtual deadlines for all HI-tasks. For LO-critical tasks, their virtual deadlines are set identical to their actual deadlines. Then, tasks are scheduled by EDF according to their *virtual* deadlines in the L-mode and HI-tasks are scheduled by EDF according to their *actual* deadlines once the system is switched to the H-mode.

B. *Algorithm* fpEDF-VD-rp

Combining fpEDF and EDF-VD, we propose algorithm fpEDF-VD-rp ("rp" stands for reserving processors) to address the new precise MC scheduling problem in this paper. fpEDF-VD-rp has two phases: a pre-processing phase and a runtime phase.

In the pre-processing phase, a scaling factor $x \in (0, 1)$ is calculated by Eq. (1). The relative virtual deadline of each HI-task τ_i is set as $x \cdot T_i$, *i.e.*, every HI-job has a virtual deadline $x \cdot T_i$ time units after its release. Furthermore, the number of processors dedicated to LO-tasks m_{LO} is calculated by Eq. (2).

$$x = \max\left\{\hat{u}_{\text{HI}}^L, \frac{2U_{\text{HI}}^L}{M^L - m_{\text{LO}} + 1}\right\} \tag{1}$$

$$m_{\text{LO}} = \begin{cases} \lceil U_{\text{LO}} \rceil, & \text{if } U_{\text{LO}} \leq 1 \\ \lceil 2U_{\text{LO}} - 1 \rceil, & \text{if } U_{\text{LO}} > 1 \end{cases} \tag{2}$$

If $m_{\text{LO}} < M^L$ and

$$\max\left\{\hat{u}_{\text{HI}}^L, \frac{2U_{\text{HI}}^L}{M^L - m_{\text{LO}} + 1}\right\} + \max\left\{\hat{u}_{\text{HI}}^H, \frac{2U_{\text{HI}}^H}{M^H - m_{\text{LO}} + 1}\right\} \leq 1, \tag{3}$$

then the pre-processing phase returns SUCCESS and enter the runtime phase; otherwise, it returns FAILURE. Please note that, by Eq. (1) and Eq. (3), it is clear that $0 < x < 1$ when fpEDF-VD-rp returns SUCCESS.

In the runtime phase, LO-tasks are scheduled in a mode-oblivious manner. The set of all LO-tasks is scheduled on m_{LO} dedicated processors by fpEDF regardless the mode. Please note that fpEDF reduces to regular uniprocessor EDF when applied on a single processor [1]. By contrast, in L-mode, HI-tasks are scheduled as a set of tasks $\bigcup_{\tau_i \in \mathcal{T}_{\text{HI}}} \left\{\left(x \cdot T_i, C_i^L\right)\right\}$ being scheduled by fpEDF on m_{HI}^L dedicated processors, where

$$m_{\text{HI}}^L = M^L - m_{\text{LO}};$$

upon a mode switch to H-mode, HI-tasks are re-scheduled as a set of tasks $\bigcup_{\tau_i \in \mathcal{T}_{\text{HI}}} \left\{\left((1-x) \cdot T_i, C_i^H\right)\right\}$ being scheduled by fpEDF on m_{HI}^H dedicated processors, where

$$m_{\text{HI}}^H = M^H - m_{\text{LO}}.$$

Note that we use a pair (T, C) to denote an ordinary (non-MC) implicit-deadline sporadic task with period T and WCET C.

C. *Schedulability Test*

We next derive and prove the schedulability test in detail.

Lemma 1. *All deadlines of* LO*-tasks are met under* fpEDF-VD-rp *scheduling if $m_{\text{LO}} < M^L < M^H$.*

Proof. Under fpEDF-VD-rp scheduling, LO-tasks are scheduled by fpEDF on m_{LO} dedicated processors and are not impacted by mode switch. By Eq. (2), it is clear that $U_{\text{LO}} \leq (m_{\text{LO}}+1)/2$. Furthermore, $\forall i, u_i \leq 1$ must hold in any feasible system. Therefore, by Thm. 1, the lemma follows, given that $m_{\text{LO}} < M^L < M^H$ ensures that indeed m_{LO} processors can devote to LO-tasks while leaving some processors for HI-tasks. □

Lemma 2. *All virtual deadlines of* HI*-tasks are met in L-mode under* fpEDF-VD-rp *scheduling, if $m_{\text{LO}} < M^L < M^H$ and*

$$x \geq \max\left\{\hat{u}_{\text{HI}}^L, \frac{2U_{\text{HI}}^L}{M^L - m_{\text{LO}} + 1}\right\}$$

Proof. In L-mode, HI-tasks are scheduled as a set of ordinary sporadic tasks $\bigcup_{\tau_i \in \mathcal{T}_{\text{HI}}} \left\{\left(x \cdot T_i, C_i^L\right)\right\}$ by fpEDF on m_{HI}^L processors. Therefore, by Thm. 1, the deadlines of these ordinary sporadic tasks, *i.e.*, the virtual deadlines of HI-tasks, are met if

$$\sum_{\tau_i \in \mathcal{T}_{\text{HI}}} \frac{C_i^L}{x \cdot T_i} \leq \frac{m_{\text{HI}}^L + 1}{2} \Leftrightarrow \frac{U_{\text{HI}}^L}{x} \leq \frac{M^L - m_{\text{LO}} + 1}{2}$$

$$\Leftrightarrow x \geq \frac{2U_{\text{HI}}^L}{M^L - m_{\text{LO}} + 1},$$

$$\text{and} \quad \forall \tau_i \in \mathcal{T}_{\text{HI}}, \frac{C_i^L}{x \cdot T_i} \leq 1 \Leftrightarrow \frac{\hat{u}_{\text{HI}}^L}{x} \leq 1$$
$$\Leftrightarrow x \geq \hat{u}_{\text{HI}}^L.$$

Thus, the lemma follows. □

Lemma 3. *All actual deadlines of* HI*-tasks are met in H-mode under* fpEDF-VD-rp *scheduling, if* $m_{\text{LO}} < M^L < M^H$ *and*

$$0 < x \leq 1 - \max\left\{\hat{u}_{\text{HI}}^H, \frac{2U_{\text{HI}}^H}{M^H - m_{\text{LO}} + 1}\right\}$$

Proof. In H-mode, HI-tasks are scheduled as a set of ordinary sporadic tasks $\bigcup_{\tau_i \in \mathcal{T}_{\text{HI}}} \left\{\left((1-x) \cdot T_i, C_i^H\right)\right\}$ by fpEDF on m_{HI}^H processors. Therefore, by Thm. 1, the actual deadlines of HI-tasks are met if

$$\sum_{\tau_i \in \mathcal{T}_{\text{HI}}} \frac{C_i^H}{(1-x) \cdot T_i} \leq \frac{m_{\text{HI}}^H + 1}{2} \Leftrightarrow \frac{U_{\text{HI}}^H}{1-x} \leq \frac{M^H - m_{\text{LO}} + 1}{2}$$
$$\Leftrightarrow 1 - x \geq \frac{2U_{\text{HI}}^H}{M^H - m_{\text{LO}} + 1}$$
$$\Leftrightarrow x \leq 1 - \frac{2U_{\text{HI}}^H}{M^H - m_{\text{LO}} + 1},$$

$$\text{and} \quad \forall \tau_i \in \mathcal{T}_{\text{HI}}, \frac{C_i^H}{(1-x) \cdot T_i} \leq 1 \Leftrightarrow \frac{\hat{u}_{\text{HI}}^H}{1-x} \leq 1$$
$$\Leftrightarrow x \leq 1 - \hat{u}_{\text{HI}}^H.$$

Thus, the lemma follows. □

Theorem 2. *The system is MCrp-schedulable by* fpEDF-VD-rp *if* $m_{\text{LO}} < M^L$ *and Eq. (3) holds.*

Proof. This theorem follows directly from the above three lemmas, noting that x is defined by Eq. (1) which implies $x > 0$ and then Eq. (3) implies $x < 1$. That is, meeting virtual deadlines implies meeting their corresponding actual deadlines as well. Please also note that m_{LO} can be easily calculated by Eq. (2) for a given task system. □

IV. Fluid Scheduling

In this section, we focus on an alternative approach, called the *dual-rate fluid* scheduling,[1] where each task τ_i is assigned a constant execution rate in each mode, denoted by θ_i^L and θ_i^H in L- and H-modes, respectively. Under fluid scheduling, all tasks conceptually progress simultaneously by "fractions" of a processor at their constant executing rates (per mode, in our particular context). Such simultaneous progression can be implemented by slicing the timeline to smaller pieces or by certain fairness based scheduling algorithms (e.g., DP-Fair [16]), which has been adapted to implement fluid scheduling for MC tasks [15].

[1]The conventional fluid scheduling assumes a single constant rate for each task, whereas two rates, *i.e.*, one rate change for each task, have been proposed and considered in the context of MC scheduling [5, 15]. Fluid scheduling with no restriction on the number of rate changes can be too general to tackle. For example, *any* actual schedule can be viewed as a fluid schedule where the rate for each task switches between 0 and 1.0.

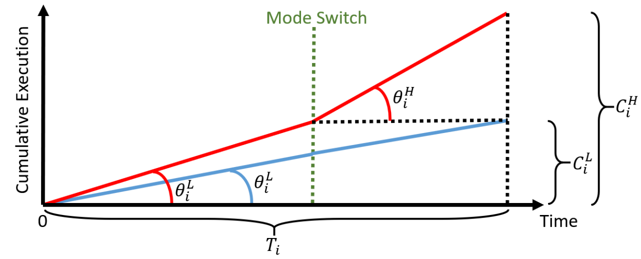


Fig. 1. An illustration of the relationship between fluid execution rates and cumulative execution over time of a task in MCF framework.

Specifically, for each LO-task, an execution speed of $\theta_i \geq u_i$ would be sufficient in both modes. By contrast, for a HI-task, it would need a speed larger than its L-mode utilization (to create sufficient gap after the mode switch to accommodate the additional execution requirement) and an even larger speed after the mode switch. Such a relationship is demonstrated in Fig. 1, where the blue line depicts the scenario for LO-tasks and the red line illustrates a representative scenario for HI-tasks.

A. *Algorithm* MCF-FR-rp

Leveraging the technique of dual-rate fluid scheduling, we propose a new algorithm, called MCF-FR-rp ("MCF" stands for mixed-criticality fluid scheduling, "FR" stands for fixed ratio, and "rp" stands for reserving processors), to address the problem considered in this paper. MCF-FR-rp consists of two steps, which are described as follows.

- A system-wide parameter λ and per-task parameters θ_i for each HI-task τ_i are calculated by:

$$\lambda = \max\left\{\frac{U_{\text{HI}}^L}{M^H - U_{\text{LO}} - U_{\text{HI}}^H + U_{\text{HI}}^L}, \max_{\tau_i \in \mathcal{T}_{\text{HI}}}\left\{\frac{u_i^L}{1 + u_i^L - u_i^H}\right\}\right\}; \quad (4)$$

$$\forall \tau_i \in \mathcal{T}_{\text{HI}}, \quad \theta_i = \frac{u_i^L}{\lambda} + u_i^H - u_i^L. \quad (5)$$

- If
$$\lambda \leq \frac{M^L - U_{\text{LO}} - U_{\text{HI}}^L}{U_{\text{HI}}^H - U_{\text{HI}}^L}, \quad (6)$$
 then each HI-task τ_i is assigned fluid execution rate $\theta_i^L = \lambda \cdot \theta_i$ in L-mode and a fluid execution rate $\theta_i^H = \theta_i$ in H-mode, whereas each LO-task τ_k is assigned a fluid execution rate $\theta_k^L = \theta_k^H = u_k^L$ in both modes,
 and return SUCCESS;

 Else return FAILURE.

Noting that $U_{\text{LO}} + U_{\text{HI}}^H \leq M^H$ and $\forall i, u_i^H \leq 1$ must hold for any feasible system, it is clear that Eq. (4) implies $0 < \lambda \leq 1$.

B. Schedulability Test

We first show that the fluid execution rates assigned by MCF-FR-rp, if they can indeed be satisfied by the underlying platform, are sufficient to ensure all deadline to be met.

Lemma 4. *If the fluid execution rates assigned by* MCF-FR-rp *are feasible, then all deadlines must be met.*

Proof. Since $\theta_k^L = \theta_k^H = u_k^L$ is assigned for each LO-task τ_k, it is clear that all deadline of LO-tasks are met.

For a HI-task τ_i, by Eq. (5), $\theta_i^L = \lambda\theta_i \geq u_i^L$ because $u_i^H \geq u_i^L$, and $\theta_i^H = \theta_i \geq u_i^H$ because $0 < \lambda \leq 1$. Therefore, HI-jobs executing entirely in either L- or H-mode must meet their deadlines.

Thus, in the rest of the proof, we only need to focus on the HI-jobs that are released before the mode switch and with a deadline after the mode switch. We consider such a job J of τ_i and let t denote its release time. If mode switch happens after time $t + \frac{C_i^L}{\theta_i^L}$, then J must have finished by time $t + \frac{C_i^L}{\theta_i^L}$ and therefore meets its deadline; otherwise, J should have triggered the mode switch at time $t + \frac{C_i^L}{\theta_i^L}$. On the other hand, because $\theta_i^L \leq \theta_i^H$, the later the mode switch, the later J completes its execution, when J needs to execute for more than C_i^L (and up to C_i^H). Therefore, the worst case for J is when the mode switch happens exactly at time $t + \frac{C_i^L}{\theta_i^L}$. In this case, J still must finish by

$$
\begin{aligned}
&t + \frac{C_i^L}{\theta_i^L} + \frac{C_i^H - C_i^L}{\theta_i^H} = t + \frac{C_i^L}{\lambda\theta_i} + \frac{C_i^H - C_i^L}{\theta_i} \\
=&t + \left(\frac{C_i^L}{\lambda} + C_i^H - C_i^L\right)\frac{1}{\theta_i} = t + \left(\frac{u_i^L}{\lambda} + u_i^H - u_i^L\right)\frac{T_i}{\theta_i} \\
=&t + \theta_i \cdot \frac{T_i}{\theta_i} = t + T_i,
\end{aligned}
$$

which is the deadline of J. So, this completes the proof and the lemma follows. □

We next show that the fluid execution rates assigned by MCF-FR-rp are indeed feasible.

Lemma 5. *It holds that* $\forall i, \theta_i^L \leq 1 \wedge \theta_i^H \leq 1$.

Proof. This is trivially true for LO-tasks. For a HI-task τ_i, by Eq. (4) we have $\lambda \geq \frac{u_i^L}{1+u_i^L-u_i^H}$, and therefore by Eq. (5) we have

$$\theta_i^H \leq \frac{u_i^L}{\frac{u_i^L}{1+u_i^L-u_i^H}} + u_i^H - u_i^L = 1.$$

Furthermore, due to $0 < \lambda \leq 1$, we then have $\theta_i^L = \lambda\theta_i = \lambda\theta_i^H \leq 1$, and the lemma follows. □

Lemma 6. *It holds that* $\sum_i \theta_i^H \leq M^H$.

Proof. By Eq. (4), we have $\lambda \geq \frac{U_{\text{HI}}^L}{M^H - U_{\text{LO}} - U_{\text{HI}}^H + U_{\text{HI}}^L}$.

Therefore,
$$
\begin{aligned}
\sum_i \theta_i^H &= \sum_{\tau_i \in \mathcal{T}_{\text{HI}}} \theta_i^H + \sum_{\tau_i \in \mathcal{T}_{\text{LO}}} \theta_i^H \\
&= \sum_{\tau_i \in \mathcal{T}_{\text{HI}}} \left(\frac{u_i^L}{\lambda} + u_i^H - u_i^L\right) + \sum_{\tau_i \in \mathcal{T}_{\text{LO}}} u_i \\
&= \frac{U_{\text{HI}}^L}{\lambda} + U_{\text{HI}}^H - U_{\text{HI}}^L + U_{\text{LO}} \\
&\leq \frac{U_{\text{HI}}^L}{\frac{U_{\text{HI}}^L}{M^H - U_{\text{LO}} - U_{\text{HI}}^H + U_{\text{HI}}^L}} + U_{\text{HI}}^H - U_{\text{HI}}^L + U_{\text{LO}} \\
&= M^H.
\end{aligned}
$$

The lemma follows. □

Lemma 7. *If Eq. (6) is true, then* $\sum_i \theta_i^L \leq M^L$ *holds.*

Proof. We have

$$
\begin{aligned}
\sum_i \theta_i^L &= \sum_{\tau_i \in \mathcal{T}_{\text{HI}}} \theta_i^L + \sum_{\tau_i \in \mathcal{T}_{\text{LO}}} \theta_i^L \\
&= \sum_{\tau_i \in \mathcal{T}_{\text{HI}}} \left(u_i^L + (u_i^H - u_i^L)\lambda\right) + \sum_{\tau_i \in \mathcal{T}_{\text{LO}}} u_i \\
&= U_{\text{HI}}^L + (U_{\text{HI}}^H - U_{\text{HI}}^L)\lambda + U_{\text{LO}},
\end{aligned}
$$

and then by Eq. (6),

$$
\begin{aligned}
&U_{\text{HI}}^L + (U_{\text{HI}}^H - U_{\text{HI}}^L)\lambda + U_{\text{LO}} \\
\leq &U_{\text{HI}}^L + (U_{\text{HI}}^H - U_{\text{HI}}^L) \cdot \frac{M^L - U_{\text{LO}} - U_{\text{HI}}^L}{U_{\text{HI}}^H - U_{\text{HI}}^L} + U_{\text{LO}} \\
= &M^L.
\end{aligned}
$$

Thus, the lemma follows. □

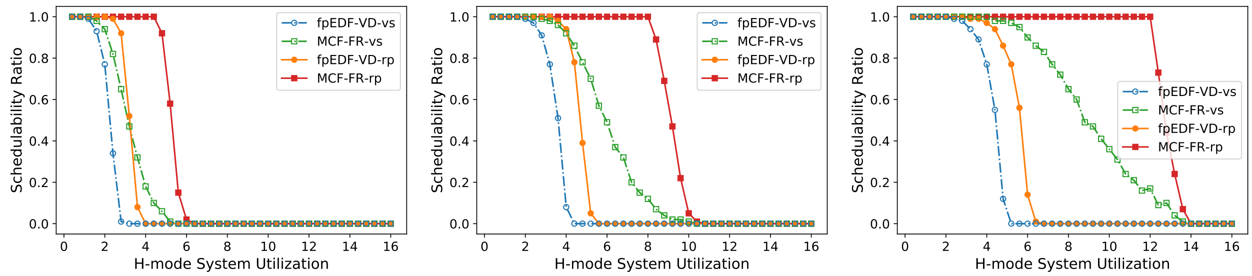
Theorem 3. *The system is MCrp-schedulable by* MCF-FR-rp *if Eq. (6) is true where* λ *is defined by Eq. (4).*

Proof. Lem. 5, Lem. 6, and Lem. 7 together imply that the fluid execution rates assigned by MCF-FR-rp are feasible if Eq. (6) holds. Therefore, by Lem. 4, this theorem follows. □

V. Evaluation

In this section, we conduct schedulability experiments to compare this work to prior work [18] on precise MC scheduling for varying-speed multiprocessors. In [18], all m processors are active in both L- and H-modes. However, the m processors must operate at a degraded speed $\rho < 1.0$ in L-mode while may run at the full-speed 1.0 in H-mode. In our experiments, we maintain the setting of $M^H = m$ and $M^L/M^H = \rho$ so that the total computing capacity of the platform, no matter in L- or H-mode, is the same for the two variants of precise MC scheduling. Then, we compare the schedulability ratios under fpEDF-VD-rp and MCF-FR-rp in this work to that under fpEDF-VD-vs and MCF-FR-vs[2] from [18].

[2] We add the "vs" suffix to the algorithms in [18] to emphasize that they are for the varying-speed model and to distinguish them from the algorithms in this paper.



(a) $M^L/M^H = \rho = 0.25$. (b) $M^L/M^H = \rho = 0.5$. (c) $M^L/M^H = \rho = 0.75$.

Fig. 2. Schedulability experiment results, where $M^H = m = 16$. Schedulability ratio is defined as the ratio of the number of schedulable task sets by the respective algorithm and the number of randomly generated task sets for each given H-mode system utilization.

Workload generation. We generate an MC task set by first generating the H-mode utilization of every task, using UUniFast-Discard [9] for given H-mode system utilization. For each task set, we mandate the first task to be a HI-task to avoid the scenario that all tasks are LO-tasks (which would be in fact non-MC and the techniques discussed in this paper should not be applied). For subsequent tasks, each of them is set to be a HI-task with probability P or a LO-task with probability $(1 - P)$. If task τ_i is a HI-task, then its L-mode utilization u_i^L is randomly chosen from $[0.2 \times u_i^H, 0.8 \times u_i^H]$. By contrast, u_i^L must equal to u_i^H for any LO-task τ_i.

Results. In Fig. 2, we report the schedulability results, where $M^H = m = 16$, $P = 0.75$, 40 tasks per task set, and $1,000$ task sets generated per given H-mode system utilization. Moreover, sub-figures (a), (b), and (c) present the results for which both M^L/M^H and ρ are 0.25, 0.5, 0.75, respectively. As seen in the figure, both fpEDF-VD-rp and MCF-FR-rp outperform their varying-speed counterpart significantly. In terms of the total number of schedulable task sets (graphically, the "area" between the plot and the x-axis), fpEDF-VD-rp is 1.36 times of its varying-speed counterpart and MCF-FR-rp is 1.5 times of its varying-speed counterpart.

VI. Conclusion

In this work, we investigated the precise scheduling of MC tasks for reserving processors in L-mode. We presented two algorithms, called fpEDF-VD-rp and MCF-FR-rp, for this new scheduling problem and provided a sufficient schedulability test for each. Our schedulability experiments demonstrated the effectiveness of the proposed algorithms and the benefits of the new scheduling model.

References

[1] Sanjoy Baruah. Optimal utilization bounds for the fixed-priority scheduling of periodic task systems on identical multiprocessors. *IEEE Transactions on Computers*, 53(6):781–784, 2004.

[2] Sanjoy Baruah, Vincenzo Bonifaci, Gianlorenzo D'Angelo, Haohan Li, Alberto Marchetti-Spaccamela, Suzanne Van Der Ster, and Leen Stougie. The preemptive uniprocessor scheduling of mixed-criticality implicit-deadline sporadic task systems. In *Proceedings of the 24th Euromicro Conference on Real-Time Systems (ECRTS)*, pages 145–154, 2012.

[3] Sanjoy Baruah, Vincenzo Bonifaci, Gianlorenzo D'Angelo, Alberto Marchetti-Spaccamela, Suzanne Van Der Ster, and Leen Stougie. Mixed-criticality scheduling of sporadic task systems. In *European Symposium on Algorithms*, pages 555–566. Springer, 2011.

[4] Sanjoy Baruah, Alan Burns, and Zhishan Guo. Scheduling mixed-criticality systems to guarantee some service under all non-erroneous behaviors. In *2016 28th Euromicro Conference on Real-Time Systems (ECRTS)*, pages 131–138. IEEE, 2016.

[5] Sanjoy Baruah, Arvind Easwaran, and Zhishan Guo. MC-Fluid: simplified and optimally quantified. In *2015 IEEE Real-Time Systems Symposium*, pages 327–337. IEEE, 2015.

[6] Ashikahmed Bhuiyan, Sai Sruti, Zhishan Guo, and Kecheng Yang. Precise scheduling of mixed-criticality tasks by varying processor speed. In *Proceedings of the 27th International Conference on Real-Time Networks and Systems*, pages 123–132, 2019.

[7] Alan Burns and Sanjoy Baruah. Towards a more practical model for mixed criticality systems. In *1st WMC*, 2013.

[8] Alan Burns and Robert Davis. Mixed criticality systems-a review. *Dept. of Computer Science, University of York, Tech. Rep*, pages 1–81, 2019.

[9] Paul Emberson, Roger Stafford, and Robert I Davis. Techniques for the synthesis of multiprocessor tasksets. In *proceedings 1st International Workshop on Analysis Tools and Methodologies for Embedded and Real-time Systems (WATERS 2010)*, pages 6–11, 2010.

[10] Rolf Ernst and Marco Di Natale. Mixed criticality systems—a history of misconceptions? *IEEE Design & Test*, 33(5):65–74, 2016.

[11] Zhishan Guo, Kecheng Yang, Sudharsan Vaidhun, Samsil Arefin, Sajal K Das, and Haoyi Xiong. Uniprocessor mixed-criticality scheduling with graceful degradation by completion rate. In *2018 IEEE Real-Time Systems Symposium (RTSS)*, pages 373–383. IEEE, 2018.

[12] Kai Huang, Luca Santinelli, Jian-Jia Chen, Lothar Thiele, and Giorgio C Buttazzo. Applying real-time interface and calculus for dynamic power management in hard real-time systems. *Real-Time Systems*, 47(2):163–193, 2011.

[13] Pengcheng Huang, Pratyush Kumar, Georgia Giannopoulou, and Lothar Thiele. Run and be safe: Mixed-criticality scheduling with temporary processor speedup. In *Design, Automation & Test in Europe Conference & Exhibition (DATE), 2015*, pages 1329–1334. IEEE, 2015.

[14] Mathieu Jan, Lilia Zaourar, and Maurice Pitel. Maximizing the execution rate of low criticality tasks in mixed criticality system. *1st WMC*, 2013.

[15] Jaewoo Lee, Kieu-My Phan, Xiaozhe Gu, Jiyeon Lee, Arvind Easwaran, Insik Shin, and Insup Lee. MC-Fluid: Fluid model-based mixed-criticality scheduling on multiprocessors. In *2014 IEEE Real-Time Systems Symposium*, pages 41–52. IEEE, 2014.

[16] Greg Levin, Shelby Funk, Caitlin Sadowski, Ian Pye, and Scott Brandt. Dp-fair: A simple model for understanding optimal multiprocessor scheduling. In *2010 22nd Euromicro Conference on Real-Time Systems*, pages 3–13. IEEE, 2010.

[17] Di Liu, Jelena Spasic, Nan Guan, Gang Chen, Songran Liu, Todor Stefanov, and Wang Yi. Edf-vd scheduling of mixed-criticality systems with degraded quality guarantees. In *2016 IEEE Real-Time Systems Symposium (RTSS)*, pages 35–46. IEEE, 2016.

[18] Tianning She, Sudharsan Vaidhun, Qijun Gu, Sajal K Das, Zhishan Guo, and Kecheng Yang. Precise scheduling of mixed-criticality tasks on varying-speed multiprocessors. In *Proceedings of the 29th International Conference on Real-Time Networks and Systems*, 2021.

[19] Steve Vestal. Preemptive scheduling of multi-criticality systems with varying degrees of execution time assurance. In *28th IEEE International Real-Time Systems Symposium (RTSS)*, pages 239–243. IEEE, 2007.

[20] Kecheng Yang, Ashikahmed Bhuiyan, and Zhishan Guo. F2VD: Fluid rates to virtual deadlines for precise mixed-criticality scheduling on a varying-speed processor. In *2020 IEEE/ACM International Conference On Computer Aided Design (ICCAD)*, pages 1–9. IEEE, 2020.