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Saving Energy by Adjusting Transmission Power in Wireless Sensor Networks

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***Abstract*—Wireless sensor networks (WSNs) have attracted a great deal of study due to the low cost of sensors and their wide range of applications. Most of the sensors used so far are point sensors which have disc-shaped sensing and communication areas. Energy-efficient communication is an important issue in WSNs because of the limited power resource and the inconvenience to recharge sensor batteries frequently. In this paper, we propose distributed algorithms to reduce communication energy consumption in WSNs by minimizing the total transmission power of sensors while maintaining the connectivity of the network. We first develop a distributed algorithm called DTRNG (Determine the Transmission power using RNG) based on RNG (Relative Neighborhood Graph) to let each sensor determine its transmission power. Then we point out that although RNG can maintain the connectivity of the network, it is not adequate to minimize the total transmission power of sensors. So we enhance it to algorithm DTCYC (Determine the Transmission power by removing the largest edge in CYCles). Mathematical proofs show that the result of the DTCYC algorithm is a minimal spanning tree, which can not only minimize the total sensor transmission power but maintain the connectivity of the network as well. Therefore, DTCYC algorithm is efficient in saving energy and can thus prolong the lifetime of WSNs.**

***Index Terms*—energy-efficient, minimal spanning tree, relative neighborhood graph, transmission power, wireless sensor networks**

I. INTRODUCTION

Wireless sensor networks (WSNs) provide a new class of computer systems and expand human ability to remotely interact with the physical world. Most of the sensors used so far are point sensors which have disc-shaped sensing and communication areas.

In this paper, we discuss energy-efficient communication in WSNs. Saving energy is very important in WSNs because of the limited power supply of sensors and the inconvenience to recharge their batteries. We propose methods to reduce communication energy by minimizing the total sensor transmission power. That is, instead of transmitting using the maximum possible power, sensors can collaboratively determine and adjust their transmission power to reach minimum total transmission power and define the topology of the WSN by the neighbor relation under certain criteria. This is in contrast to the "traditional" network in which each node transmits using its maximum transmission power and the topology is built implicitly without considering the power issue.

Choosing the right transmission power critically affects the system performance in several ways. First, it affects network spatial reuse and hence the traffic carrying capacity [4]. Choosing too large a power level results in excessive interference, while choosing too small a power level results in a disconnected network. Second, it impacts on the contention for the medium. Collisions can be mitigated as much as possible by choosing the smallest transmission power subject to maintaining network connectivity [7], [9].

Hence, in this paper, our goal is to find distributed methods to let each sensor decide its transmission power by communicating with other sensors to minimize total sensor transmission power while maintaining the connectivity of the network. We first put forward a distributed algorithm called DTRNG based on the *Relative Neighborhood Graph* (RNG) [10] in graph theory. We point out that it can maintain the network connectivity, but may not minimize the total sensor transmission power. Then we enhance it to the DTCYC algorithm, whose basic idea is to let each sensor remove the largest edge in every cycle involving it as a vertex. Mathematical proofs show that it can not only maintain the network connectivity but also minimize the total transmission power. Since sensors decide their transmission powers by exchanging information with other sensors, our algorithms can be applied not only to static sensor networks but also to mobile sensor networks where sensors move due to natural phenomena such as wind, mudflows, etc. They can also be applied to ad hoc wireless networks where nodes move more frequently. In summary, using our algorithms, a node can dynamically adjust its transmission power if it is involved in a topological change.

The rest of the paper is organized as follows: Section II summarizes the related work, Section III is the preliminary, Section IV formulates the problem and provides the solution, and Section V concludes the paper.

II. RELATED WORK

In the literature, several algorithms have been proposed to reduce energy consumption by adjusting sensor transmission power. It is used by [1], [11] to minimize energy in broadcast communication and by [5], [8] to do topology control.

Wieselthier et al. define in [11] a topology control algorithm using node's adjustable transmission power based on *Minimum*

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Spanning Tree (MST). It is designed in a global manner, meaning that each node needs global network information. The authors also propose in [11] two other globalized algorithms: BLU and BIP to minimize broadcast energy consumption by adjusting node's transmission power. The BLU (Broadcast Least-Unicast-cost) applies the Dijkstra's algorithm and the BIP (Broadcast Incremental Power) is a modified version of the Prim's algorithm. Adjustable transmission power is also used by Cartigny et al. in [1] to build RNG locally to solve the minimum-energy broadcast problem.

Several localized solutions exist based on local spanning subgraphs using a node's adjustable transmission power to manage network topology, such as in MST [5] and SPT [8]. In [5], the network topology is constructed by each node building its local MST independently (with the use of information locally collected) and only keeping one-hop on-tree nodes as neighbors. It proves that the topology resulted preserves the network connectivity. Rodoplu et al. [8] introduce the notion of relay region and enclosure for the purpose of power control. It is shown that the network is strongly connected if every node maintains links with the nodes in its enclosure and the resulting topology is a minimum power topology.

Different from these works, in this paper, we design distributed algorithms to reduce communication energy consumption in WSNs by minimizing the total transmission power of sensors while keeping the connectivity of the network. We start from designing an algorithm using the RNG graph and then extend it to provide an optimal solution to our problem.

III. Preliminary

We assume that in a two-dimensional planar world, a sensor can detect objects within its sensing region which is modeled as a disc with a radius s. It can also send messages to other sensors using its transmission power p which decides the maximum distance its messages can reach other sensors based on p. This maximum distance is called the *transmission range* r of the sensor and the resultant area is also a disc (see Fig. 1). In this paper, we assume all the sensors deployed are of the same type. Each sensor has a maximum transmission power level which results in a maximum transmission range. A common model to calculate the energy consumption of a sensor u if it uses transmission range $r(u)$ to transmit a unit message is: $E(u) = r(u)^{\alpha}$ [2], [3], [6], [11]. Here, α is a constant greater than 2. Thus, the smaller the $r(u)$, the more energy can be saved. So if u can reach another sensor using a smaller $r(u)$, then in order to save energy, it is not necessary for u to transmit with its full power level.

Therefore, based on the transmission power level each sensor uses, a WSN can be modeled as a graph $G = (V, E)$, where V is the set of all sensor nodes and E is the set of all edges between pairs of sensor nodes. If two sensor nodes are within each other's transmission range, there is an edge between them in G which means they can communicate directly and they are called *neighbors*. Otherwise, they have to rely on intermediate nodes between them to relay messages.

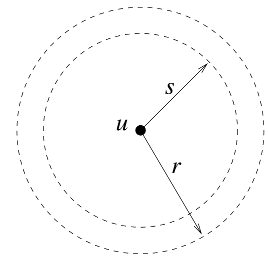
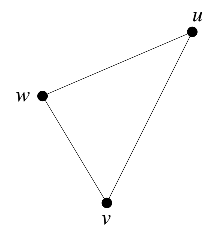
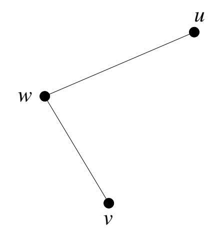


Fig. 1. Sensor model



(a) A triangle



(b) The RNG of the triangle

Fig. 2. A triangle and its RNG

If the distance between nodes A and B is $|AB|$, in order to reach B, the minimum transmission power level that A uses should cover the distance $|AB|$. So we label this power level as $p_{|AB|}$. We also assume no sensor is unconnected from the rest of the WSN at its maximum transmission power.

IV. Problem formulation and our solution

Now the problem to be solved in this paper can be stated as: Given a WSN, assume the sensors can adjust their transmission powers p_1, p_2, $\cdots$, p_n, minimize $\sum_{i=1}^{n} p_i$ while maintaining the connectivity of the network.

In this section, we put forward distributed algorithms for a sensor to decide its transmission power without affecting the connectivity of the network. We first develop a distributed algorithm called DTRNG (*Determine the Transmission power using RNG*) based on RNG. Then we point out its drawback and enhance it to algorithm DTCYC (*Determine the Transmission power by removing the largest edge in CYCles*). Mathematical proofs show that the DTCYC algorithm not only minimizes the total sensor transmission power but also maintains the connectivity of the network.

A. Determine transmission power using RNG

Our first algorithm was inspired by the RNG [10] in graph theory. RNG is an undirected graph defined on a set of points in the Euclidean plane by connecting two points u and v by an edge whenever there does not exist a third point w that is closer to both u and v than they are to each other. In other words, if there exists such a w, points u and v should not be connected. For a simple example, if there is a triangle uvw as shown in Fig. 2 and uv is the largest edge in the triangle, the RNG of the triangle contains only two edges: uw and vw. RNG has a nice property as stated in Theorem 1.

Theorem 1: Given a weighted graph $G = (V, E)$, $RNG(G)$ contains an $MST(G)$ as a subgraph.

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Algorithm DTRNG: Determine the Transmission power using RNG to minimize total transmission power

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1: Each sensor u calls Algorithm DTNBOR to determine the minimum transmission power to reach each of its neighbors.
2: **repeat**
3: Each sensor u checks the following condition:

$$p_{|uw|} \leq p_{|uv|} \text{ and } p_{|vw|} \leq p_{|uv|} \quad \forall w \in N(u) \cup N(v) \tag{1}$$

4: If it is true, sensor u will remove v and the transmission power to reach v from its neighbor table.
5: **until** there are no more removals.
6: Each sensor will use $p_{|largest\ edge\ incident\ on\ it|}$ as its transmission power.

Fig. 3. Algorithm DTRNG

Proof: If there is a triangle in a weighted graph G, the triangle can be looked as a special case of a cycle. Based on later Lemma 1 in this paper, the largest edge in a cycle cannot be in $MST(G)$. On the other hand, to generate an RNG, the largest edge in every triangle in G is removed. Therefore, $MST(G)$ is a subgraph of $RNG(G)$. □

Using this property of RNG, the distributed algorithm for a sensor to decide its transmission power can be written in Algorithm DTRNG (see Fig.3). This algorithm first calls Algorithm DTNBOR (see Fig.4) to find all the neighbors of a sensor and the different minimum transmission powers it uses to reach them. In Algorithm DTNBOR, a sensor first sends out a Hello message with its maximum transmission power. If it gets a REPLY from another sensor, that means they are neighbors. Next it will repeatedly reduce the transmission power to reach the neighbor until it can no longer hear from it before the timer expires. Then the minimum transmission power to reach the neighbor is known. It should be noted that δ in the algorithm decides the precision of the minimal transmission power. The smaller it is, the more accurate the minimal transmission power is, but the longer the algorithm will run before it terminates.

After each sensor has built its neighbor table, it can generate the RNG of the network by checking condition (1) at Step 3 in Algorithm DTRNG. In condition (1), $p_{|xy|}$ indicates the minimum transmission power to connect sensors x and y. $N(x)$ is the neighbor set of x, and w is any other sensor in the union of u and v's neighbors. The meaning of condition (1) is that an edge uv will not be included in the topology when there exists a neighbor w such that both $p_{|uw|} \leq p_{|uv|}$ and $p_{|vw|} \leq p_{|uv|}$ are true. In other words, edge uv will be removed if it is the largest edge in triangle uvw. As proved by Theorem 1, RNG contains an MST of the network as a subgraph. So it is guaranteed that the resultant topology is connected. Thus, each sensor can use transmission power level $p_{|largest\ edge\ incident\ on\ it|}$ to just cover the largest edge incident on it in RNG to transfer messages.

Algorithm DTNBOR: Determine the minimum Transmission power to reach each NeighBOR

1: Each sensor needs to build its neighbor table which contains the IDs of its neighbors and the transmission powers to reach them. Initially, the IDs and the transmission powers are empty.
2: Each sensor u starts a timer and sends out a Hello message containing its ID and its neighbor table using its maximum transmission power.
3: **repeat**
4: If a sensor v receives a Hello message from sensor u, it will add u to its neighbor table and record the transmission power to reach it, and send out a REPLY message containing its ID and its neighbor table.
5: If sensor u receives a REPLY message from a sensor v, it will add v to its neighbor table and update the transmission power to reach it. Then u will reduce its transmission power level to v by δ, start a timer and send out a Hello message to v containing its ID and its neighbor table using the reduced transmission power.
6: If sensor u does not hear from v before the timer expires, it will use the current transmission power recorded in its neighbor table as the minimum transmission power to reach v.
7: **until** there are no more changes in each sensor's neighbor table

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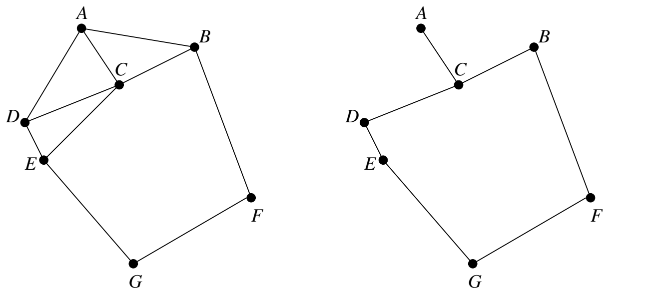
Fig. 4. Algorithm DTNBOR

We use an example to explain the DTRNG algorithm. Suppose there is a network of sensors connected as in Fig. 5(a). Using the DTNBOR algorithm, each sensor builds its neighbor table containing the IDs of its neighbors and the minimum transmission powers to reach them. For example, sensor A finds three neighbors $\{B, C, D\}$ and the minimum transmission powers to reach them are: $\{p_{|AB|}, p_{|AC|}, p_{|AD|}\}$. Next each sensor will remove the largest edge in every triangle having it as a vertex. The resultant graph is an RNG of the original graph (see Fig. 5(b)). And then each sensor can decide its transmission power based on the largest edge incident on it in the RNG graph. For example, sensor C has three edges AC, BC and DC incident on it, in which, DC is the largest. So sensor C will use $p_{|DC|}$ as its transmission power level.

B. Drawback of the RNG method

The essence of the RNG method is to adjust the transmission power of sensors by removing the largest edge in every triangle in RNG. Even so, it still may not produce the minimal total sensor transmission power. For example, in Fig. 6, using the RNG method, the largest edge AC in triangle ABC is removed. Then for sensor A, it has AE and AB incident on it, of which AE is longer. So it will use $p_{|AE|}$ as its transmission power. Similar to A, sensor E will use $p_{|AE|}$ as its transmission power. However, if AE is also removed, the graph is still connected and A can use $p_{|AB|}$ and E can use



(a) A sensor network (b) The RNG of the sensor network

Fig. 5. Decide each sensor's transmission power by constructing an RNG

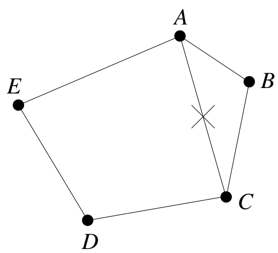


Fig. 6. An example of the drawback of the RNG method

$p_{|ED|}$ to further reduce the total transmission power of sensors. Therefore, the RNG method can maintain the connectivity of the network, but may not minimize $\sum_{i=1}^{n} p_i$. As we take another look at the figure, AE is the longest edge in the cycle $ABCDEA$. So if we remove the longest edge in a cycle, or every cycle, can we get better results? This is what we will address in the next section.

C. Determine transmission power by removing the largest edge in cycles

As we know, given a random weighted graph $G = (V, E)$, the RNG method can be looked as each sensor remove the largest edge of every triangle that includes the sensor as a vertex. Now, we can extend this idea to let a sensor remove the largest edge in every cycle that includes it as a vertex. Thus, we develop Algorithm DTCYC (see Fig. 7) to determine the transmission power of each sensor using this idea.

As in Algorithm DTRNG, Algorithm DTCYC first calls Algorithm DTNBOR to build the neighbor table for each sensor. Next, there are two parts in Algorithm DTCYC. In the first part, each sensor needs to find out if there are any cycles involving it as a vertex. In the second part, it needs to remove the largest edge in every cycle involving it as a vertex. We use the same example in Algorithm DTRNG to explain this algorithm. The same sensor network is shown in Fig. 8(a). After each sensor builds its neighbor table using the DTNBOR algorithm, it tries to find out if it is involved in any cycle. If it is, it will remove the largest edge in every cycle. For example, sensor B removes edges AB and BF because AB is the largest edge in triangle ABC and BF is the largest edge in cycle $BCEGFB$. All the other sensors will do the same thing. Eventually the resultant subgraph generated is an

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Algorithm DTCYC: Determine the Transmission power by removing the largest edge in CYCles to minimize total transmission power

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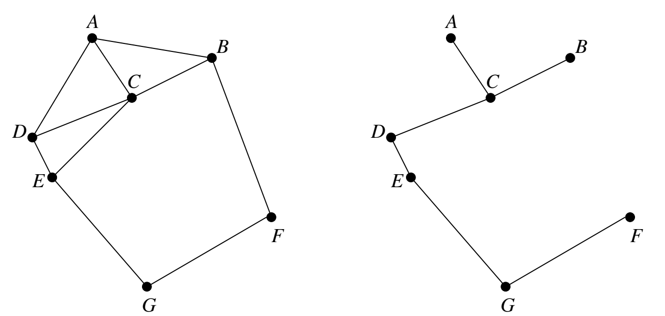
1: Each sensor u calls Algorithm DTNBOR to find the minimum transmission power to reach each of its neighbors.
2: **repeat**
3: Each sensor u starts a timer and broadcasts a 'Find_cycle' message including its ID using the maximum of all the minimum transmission powers to reach its neighbors obtained from Step 1.
4: If a sensor v gets a 'Find_cycle' message, it will relay the message by adding its ID and the transmission power to reach it from its direct sender to all its neighbors.
5: Before the timer expires, if u receives its own 'Find_cycle' message and the path containing all the relay nodes and the transmission power between each pair, then it knows it is involved in a cycle.
6: Based on the information u gets, u can find the largest edge in the cycle. Then it will send a 'Remove_edge' message including the two vertices of that edge to the next sensor on the cycle path.
7: If a sensor v receives the 'Remove_edge' message and if it is one of the vertices of that edge and it has an edge to the other sensor w on that edge, it will remove that edge by removing w and the transmission power to reach it from its neighbor table; otherwise, it just relays the message on.
8: **until** there are no more changes in each sensor's neighbor table.
9: Each sensor will use $p_{|largest\ edge\ incident\ on\ it|}$ as its transmission power.

Fig. 7. Algorithm DTCYC



(a) A sensor network (b) The MST of the sensor network

Fig. 8. Determine each sensor's transmission power by removing the largest edge in cycles

MST of the original topology as is proved by Lemma 2 and is shown in Fig. 8(b). After the MST is constructed, each sensor can use the transmission power level to just cover the largest edge incident on it to transfer messages. Comparing with the DTRNG algorithm, the DTCYC algorithm can further reduce the overall transmission power of sensors. In this example, sensor B only removes edge AB in the DTRNG algorithm, but it also removes edge BF in the DTCYC algorithm. So instead of using the transmission power level to just cover $|BF|$ in the RNG graph, now it can use a smaller power level to just cover $|BC|$. The following lemmas and theorems prove that Algorithm DTCYC is a solution to our problem. The MST generated guarantees that the sensors are connected and $\sum_{i=1}^{n} p_i$ is minimized. Also note that the order of removing the largest edge in every cycle will not affect the minimization of $\sum_{i=1}^{n} p_i$ since the resultant graph is an MST.

Lemma 1: Given a weighted graph $G = (V, E)$, the largest edge e in any cycle $\notin MST(G)$.

Proof: This can be proved by contradiction. Without loss of generality, suppose the largest edge e in any cycle $e_1 e_2 \cdots e_n$ is in $MST(G)$. To make a tree, one edge in the cycle except e needs to be removed. Since e is the largest edge in the cycle, removing e will result in a spanning tree smaller than the one removing any other edge in the cycle. Therefore, the spanning tree without e is smaller than the one with e. So the spanning tree containing e cannot be the minimum. This conflicts with the assumption and proves the lemma. □

Lemma 2: Given a weighted graph $G = (V, E)$, after removing the largest edge e in every cycle, the resultant graph is an $MST(G)$.

Proof: This theorem can be proved by induction. The base case is that there are n vertices in the graph and the number of edges discovered by Algorithm DTCYC is $n - 1$. This is the MST of the graph. Now assume that the theorem is true to a graph with n vertices and the number of edges is less than num_e. Next we prove that the theorem is still true for a graph with n vertices and num_e number of edges. Starting from this graph, a list of largest edges are removed from the cycles. Suppose e_1 is removed from cycle c_1, e_2 is removed from cycle c_2, $\cdots$, and e_k is removed from c_k. From Lemma 1, the largest edge e_1 in cycle c_1 cannot be in the MST of the graph. After e_1 is removed, the remaining graph has n vertices and less than num_e number of edges. According to the assumption, the tree generated by removing the largest edge in every cycle in the remaining graph is an MST of G. This proves the theorem. □

Theorem 2: Given a WSN, assume the sensors can adjust their transmission powers p_1, p_2, $\cdots$, p_n, the DTCYC algorithm can minimize $\sum_{i=1}^{n} p_i$ while maintaining the connectivity of the network.

Proof: According to the DTCYC algorithm, the largest edge in every cycle involving any sensor vertex is removed and each sensor can adjust its transmission power to just cover the largest edge incident on it in the resultant graph. From Lemma 2, the resultant graph of the WSN is an MST. The MST is a subgraph of the originally graph which is connected, involves all the vertexes, and makes $\sum_{i=1}^{n} p_i$ minimal. □

Theorem 3: Given a WSN with n nodes, in the worst case, the DTCYC algorithm will terminate after removing $\frac{n^2}{2} - \frac{3}{2}n + 1$ edges.

Proof: In a WSN with n nodes, the maximum number of edges is $\frac{n(n-1)}{2}$ in a complete graph. The number of edges in the resultant MST is $n - 1$. Therefore, in the worse case, DTCYC algorithm will remove $\frac{n(n-1)}{2} - (n-1) = \frac{n^2}{2} - \frac{3}{2}n + 1$ edges before it terminates. □

V. Conclusion

In this paper, we proposed algorithms to reduce communication energy consumption in WSNs by minimizing the total transmission power of sensors while keeping the connectivity of the network. We first developed a distributed algorithm DTRNG based on RNG to let each sensor determine its transmission power. Then we pointed out that although RNG can maintain the connectivity of the network, it is not adequate to minimize the total transmission power of sensors. So we enhanced it to algorithm DTCYC. Mathematical proofs showed that the result of the DTCYC algorithm is an MST, which can not only minimize the total sensor transmission power but maintain the connectivity of the network as well. Therefore, the DTCYC algorithm is efficient in saving energy and can thus prolong the lifetime of the network. In the future, we will conduct simulations to verify our results.

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